Output Consensus of Heterogeneous Linear Multi-Agent Systems by Event-Triggered Control

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OUTLINES

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CONTENTS

Multi-Agent System (MAS)

- Modeling/describing some collective behaviors of some animals.
- Wide applications in engineering problems.

Some engineering applications of the MAS.
(http://www.ri.cmu.edu/research_guide/multi_agent_systems.html)

- One fundamental problem: consensus problem.

Two Kinds of MASs

- Early researches focused on MAS with identical dynamics.

$$\dot{x}_i = Ax_i + Bu_i, \;\; i = 1, \cdots, N, \tag{1.1}$$

–called homogeneous MAS.

- In many applications, the agents' dynamics are non-identical.

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i \\ y_i &= C_i x_i, \quad\quad i = 1, \cdots, N, \end{aligned} \tag{1.2}$$

–called heterogeneous MAS.

- All agents communicate with each other through a communication graph $\mathcal{G}$.

Consensus Problem

- Homogeneous MAS: state consensus problem.
  Definition: Design u_i such that

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0,\ \forall i,j = 1,\cdots,N, \tag{1.3}$$

  holds for any finite $x_i(0)$, $\forall i = 1,\cdots,N$.

- Heterogeneous MAS: output consensus problem.
  Definition: Design u_i such that

$$\lim_{t\to\infty} \|y_i(t) - y_j(t)\| = 0,\ \forall i,j = 1,\cdots,N, \tag{1.4}$$

  holds for any finite $x_i(0)$, $\forall i = 1,\cdots,N$.

- **Remark 1**: Output consensus problem includes state consensus problem as a special case.

Why Event-Triggered Strategy?

- Individual agents equipped with microprocessors and some actuation modules.
  - On-board energy and resources are limited.
  - Energy-saving control schemes are needed.
- To reduce the communication load.
  - Proposed in stabilization problem for a single system [Tabuada (2007)].
  - Better performance than traditional periodical sampling [Astrom & Bernhardsson (1999, 2002)].
  - Mainly applied to some MASs with simple agent dynamics.

Problem Statement

- Design an event-triggered control scheme, such that the output consensus problem of heterogeneous MAS (1.2) can be solved.
  - Control input u_i can only access information from itself and its neighboring agents.
  - Event-triggered strategy should be integrated in the control scheme.

CONTENTS

- Main Challenges
- Internal Reference Model
- Event-Triggered Control for Homogeneous Systems

Main Challenges

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- Heterogeneity Problem.
  - Different dynamics: $A_i \neq A_j$, $B_i \neq B_j$ for $i \neq j$.
  - Different state dimensions.
- Use of event-triggered strategy.
  - Convergence analysis.
  - Development of event-triggering condition based on local information.
  - Feasibility analysis.

Internal Reference Model

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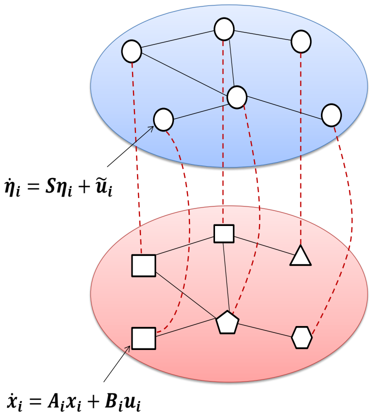
- Introduction of internal reference models.



The internal reference models of the heterogeneous MAS.

Key Idea

- Designed to be of identical dynamics.

$$\dot{\eta}_i = S\eta_i + \tilde{u}_i. \tag{2.1}$$

- Regarded as a homogeneous MAS.
  - Exchanging information: states of the internal reference models.
  - To reduce the communication load: event-triggered strategy.
- Objective: $(\eta_i - \eta_0) \to 0$ as $t \to \infty$, $\forall i = 1, \cdots, N$.
- η_0 can be regarded as a reference signal for each agent.

Event-Triggered Control for Homogeneous MAS

- For a homogeneous MAS

$$\dot{x}_i = Ax_i + B\tilde{u}_i,\ \ i = 1, \cdots, N, \tag{2.2}$$

  - Define the combined measurement as

$$\tilde{q}_i(t) = \sum_{j=1}^{N} a_{ij}\left(x_j(t) - x_i(t)\right). \tag{2.3}$$

  - Define the measurement error as

$$\tilde{e}_i(t) = \tilde{q}_i(t_k^i) - \tilde{q}_i(t). \tag{2.4}$$

- Control law for each agent:

$$\tilde{u}_i(t) = \tilde{K}\tilde{q}_i(t_k^i),\ \ t \in \left[t_k^i, t_{k+1}^i\right), \tag{2.5}$$

with $\{t_0^i, t_1^i, \cdots\}$ will be determined by the triggering condition.

- Triggering condition

$$h(\tilde{e}_i(t), \tilde{q}_i(t)) = 0. \tag{2.6}$$

A Previous Result

- **Lemma 1 [Hu et al. (2014)]**: Under the assumptions that (A, B) is stabilizable and the undirected communication graph $\mathcal{G}$ is connected, there always exists at least one solution $P > 0$ for the following inequality

$$PA + A^T P - \alpha PBB^T P + \beta I_n \leq 0, \tag{2.7}$$

where $0 < \alpha \leq 2\lambda_2$, $\beta \geq 2\lambda_N$, with λ_2 and λ_N the Fiedler eigenvalue and the largest eigenvalue of the Laplacian matrix of $\mathcal{G}$, respectively. Then, letting $\tilde{K} = B^T P$, the state consensus of homogeneous multi-agent system (2.2) can be achieved by the control law (2.5) and the following triggering condition

$$h(\tilde{e}_i(t), \tilde{q}_i(t)) = \|\tilde{e}_i(t)\| - \tilde{\gamma}_i \|\tilde{q}_i(t)\| = 0. \tag{2.8}$$

where $\tilde{\gamma}_i = \sqrt{\frac{\sigma_i \cdot a(2-a\rho)}{\rho}}$ with $\sigma_i \in (0, 1)$, $\rho = \left\|PBB^T P\right\|$, and a being a positive number satisfying $a < \frac{2}{\rho}$.

CONTENTS

Event-Triggered Control Scheme

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- Output feedback controller.

$$\begin{aligned} \dot{\eta}_i &= S\eta_i + K\sum_{j=1}^{N} a_{ij}\left(\eta_j(t_k^i) - \eta_i(t_k^i)\right), \ t \in \left[t_k^i, t_{k+1}^i\right) \\ \dot{\xi}_i &= A_i\xi_i + B_i u_i + H_i\left(C_i\xi_i - y_i\right) \\ u_i &= K_{1i}\xi_i + K_{2i}\eta_i, \ i = 1, \cdots, N. \end{aligned} \tag{3.1}$$

Design parameters: S, K, H_i, K_{1i}, K_{2i}; Triggering time sequence: $\{t_0^i, t_1^i, \cdots\}$.

- Event-triggering condition.

$$h(e_i(t), q_i(t)) = \|e_i(t)\| - \gamma_i \|q_i(t)\| = 0, \tag{3.2}$$

where $q_i(t) = \sum_{j\in\mathcal{N}_i}(\eta_j(t) - \eta_i(t))$, $e_i(t) = q_i(t_k^i) - q_i(t)$ and γ_i can be calculated by utilizing Lemma 1.

- Questions: Under what condition, the problem can be solved by (3.1) and (3.2)? How to design those parameters?

Assumptions & Result

- Assumptions:
  - Each pair of (A_i, B_i) is stabilizable.
  - Each pair of (A_i, C_i) is detectable.
  - The undirected communication graph $\mathcal{G}$ is connected.

- **Theorem 1**: Consider the heterogeneous linear multi-agent system (1.2) under Assumptions 1-3. The output consensus problem can be solved by the proposed controller (3.1) with the triggering condition (3.2) if and only if there exists (S, R), such that the following equations have solutions (Π_i, Γ_i) for $i = 1, \cdots, N$, where S, R, Π_i and Γ_i all have compatible dimensions,

$$A_i\Pi_i + B_i\Gamma_i = \Pi_i S. \tag{3.3}$$

$$C_i\Pi_i = R. \tag{3.4}$$

Notes

- **Remark 2**: Internal reference model is used to generate a virtual reference signal η_0 for each agent.
- The dynamics and output of the virtual reference signal

$$\begin{aligned} \dot{\eta}_0(t) &= S\eta_0(t) \\ y(t) &= R\eta_0(t) \end{aligned}$$

with $(y_i(t) - y(t)) \to 0$ as $t \to \infty$, $\forall i = 1, \cdots, N$.

Parameters Design

- Design procedure.
  - Step 1: Choose proper matrices S and R, such that (3.3) and (3.4) have solution pairs (Π_i, Γ_i), $i = 1, \cdots, N$.
  - Step 2: Choose proper Λ_i, such that $A_i + B_i\Lambda_i$ is Hurwitz. Let K_{1i}, K_{2i} be as follows,

$$K_{1i} = \Lambda_i, K_{2i} = \Gamma_i - \Lambda_i\Pi_i. \tag{3.5}$$

  - Step 3: Choose a proper H_i, such that $A_i + H_iC_i$ is Hurwitz.
  - Step 4: Let $K = P$, where $P > 0$ satisfies the following inequality

$$PS + S^TP - \alpha PP + \beta I_m \leq 0. \tag{3.6}$$

  - Step 5: Let $t_0^i, t_1^i, \cdots$, $\forall i = 1, \cdots, N$, to be determined by the proposed triggering condition (3.2).

Exclusion of Singular Triggering

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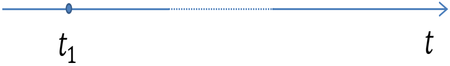
- Singular Triggering.
  - Definition:

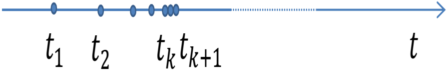


  - Method: To prove that if t_k^i exists and $q_i\left(t_k^i\right) \neq 0$, the next triggering time t_{k+1}^i exists and $q_i\left(t_{k+1}^i\right) \neq 0$.
  - Requirement: The proof holds for any agent.

- **Corollary 1**: Consider the heterogeneous multi-agent system (1.2) and the control scheme (3.1). No agent will exhibit singular triggering behavior.

Exclusion of Zeno behavior

- Zeno behavior.
  - Definition:



  - Method: To prove that the length of inter-event interval is strictly positive.
  - Requirement: The proof holds for any agent.

- **Corollary 2**: Consider the heterogeneous multi-agent system (1.2) and the control scheme (3.1). No agent will exhibit Zeno behavior.
  - Calculate the inter-event interval for ith agent

$$t_{k+1}^i - t_k^i > \frac{1}{\|S\|} \ln \left(\frac{\|S\| \, s_k^i}{\alpha_k^i} + 1 \right) \geq 0. \tag{3.7}$$

where s_k^i and α_k^i are two positive constants.

Drawback of Event-Triggered Strategy

- In the proposed event-triggering condition

$$h(e_i(t), q_i(t)) = \|e_i(t)\| - \gamma_i \|q_i(t)\| = 0,$$

continuous monitoring of $e_i(t)$ and $q_i(t)$ are required.

- How to avoid this constraint?

Self-Triggered Strategy

- Main idea: Estimate the next triggering time based on the measurement at previous triggering time.
- We propose the following condition

$$\|e_i(t)\| \leq \frac{\gamma_i}{\sqrt{2+2\gamma_i^2}} \left\| q_i\left(t_k^i\right) \right\| = s_k^i$$

which implies $h\left(e_i(t), q_i(t)\right) \leq 0$.

- Calculate the time it will be needed for $\|e_i(t)\|$ to increase to s_k^i.

Self-Triggering Condition

- Calculate the increasing rate of $\|e_i(t)\|$.

$$\frac{d}{dt}\|e_i(t)\| \leq \|S\| s_k^i + w_i(t),$$

where $w_i(t) = \left\| (d_i P - S) q_i(t_k^i) - P \sum_{j \in \mathcal{N}_i} q_j(t^j_{k'(t)}) \right\|$, with $t^j_{k'(t)}$ being the latest triggering time for agent j, $j \in \mathcal{N}_i$.

- Self-Triggering Rule

If no neighboring agent is triggered ahead of agent i, then $t_{k+1}^i = \frac{s_k^i}{\|S\| s_k^i + w_i(t_k^i)}$.
Otherwise, if one neighboring agent j is triggered first at time $t^{'}$, then update $w_i(t)$ as $w_i(t^{'})$ and calculate the left time which will be needed for $\|e_i(t)\|$ to increase to s_k^i.

- The feasibility of self-triggering rule: omitted here.

Result

- **Theorem 2**: Consider the heterogeneous linear multi-agent system (1.2) under Assumptions 1-3. The output consensus problem can be solved by the proposed controller (3.1) and the self-triggering rule if and only if there exists (S, R), such that (3.3) and (3.4) always have solutions (Π_i, Γ_i) for $i = 1, \cdots, N$, where S, R, Π_i and Γ_i all have compatible dimensions.

AN EXAMPLE

- System Model
- Simulations

System Model

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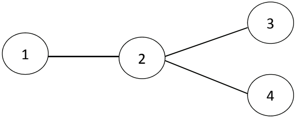
- Consider the following heterogeneous MAS [Wieland et al. (2011)].

$$\dot{x}_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & c_i \\ 0 & -d_i & -a_i \end{pmatrix} x_i + \begin{pmatrix} 0 \\ 0 \\ b_i \end{pmatrix} u_i$$

$$y_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x_i,\ i = 1, 2, 3, 4,$$

where parameters $\{a_i, b_i, c_i, d_i\}$ are set as $\{1, 1, 1, 0\}$, $\{10, 2, 1, 0\}$, $\{2, 1, 1, 10\}$ and $\{2, 1, 1, 1\}$, respectively.

- The communication graph.



The communication graph $\mathcal{G}$.

Control Design

- Event-triggered control scheme is designed according to Steps 1-5.
  - In Step 1:

$$S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

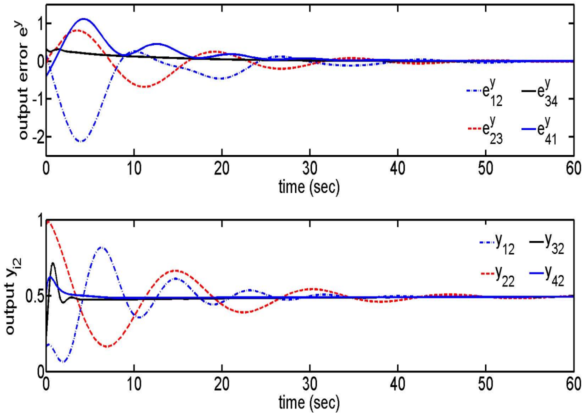
  - In Step 2:

$$\Lambda_i = \begin{pmatrix} -1 & -1 & -1 \end{pmatrix},$$
$$K_{1i} = \Lambda_i = \begin{pmatrix} -1 & -1 & -1 \end{pmatrix},$$
$$K_{2i} = \Gamma_i - \Lambda_i \Pi_i = \begin{pmatrix} 1 & 1 + \frac{d_i}{b_i} \end{pmatrix}.$$

  - In Step 3: We set $H_i = \begin{bmatrix} 0 & 0 \\ -10 & -10 \\ 9 & 9 \end{bmatrix}, i = 1, \cdots, 4.$
  - In Step 4: $K = P = \begin{pmatrix} 1.9848 & 0.2462 \\ 0.2462 & 2.0459 \end{pmatrix}.$
  - In Step 5: Threshold in the triggering condition can be calculated $\gamma_i = 0.1942$. Triggering time sequence can be calculated by event-triggering condition or self-triggering rule.

Simulations

- Simulation results.



Output response of all agents via event-triggered control scheme.

Note: $y_i = \mathrm{col}(y_{i1}, y_{i2})$ and $e^y_{ij} = y_{i1} - y_{j1}$.

Simulations Continued

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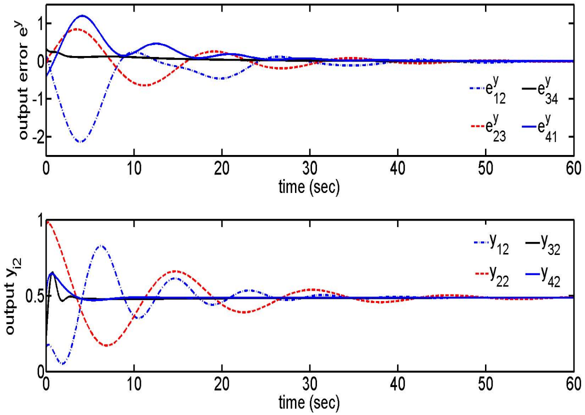
- Simulation results.



Output response of all agents via self-triggered control scheme.

Note: $y_i = \mathrm{col}(y_{i1}, y_{i2})$ and $e^y_{ij} = y_{i1} - y_{j1}$.

Simulations Continued

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- Performances comparison between two proposed control schemes

Control scheme	T_s (sec)	Triggering numbers for agents			
		1	2	3	4
Event-triggered	191.97	249	675	248	39
Self-triggered	85.08	805	970	421	415

where T_s is defined as a minimum time, such that, $\|y_i(t) - y_j(t)\| \leq 0.001$ when $t \geq T_s$, for any agents i, j.

- Less settling time while more triggering numbers are needed to reach output consensus by the self-triggered control scheme.

CONTENTS

Conclusions

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- Output consensus problem of heterogeneous linear MASs has been studied.
- A novel event-triggered control scheme has been proposed.
- Feasibility of the proposed control scheme has been discussed.
- A novel self-triggered control scheme has been proposed.

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Thank You

- **Thank You!**