Distributed Coordination of Networked Euler-Lagrange Systems

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Outline

1. Preliminaries and Problem Statement
   - Motivation
   - Some Research Directions

Outline

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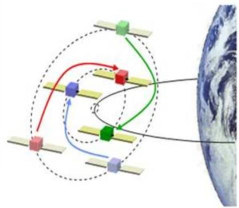
Preliminaries and Problem Statement

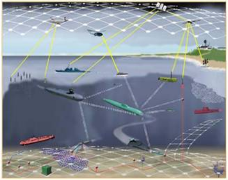
Distributed Control of Networked Multi-agent Systems - Motivation

Biological examples: flocks of birds, schools of fish, colonies of bacteria, and swarms of ants

Computer graphics: boids

Local interaction (no information relay) $\Longrightarrow$ collective group behavior





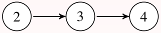
Navy UUV Master Plan

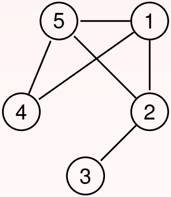
Objective: design distributed control algorithms for networked engineered systems with only local interaction

Representation of Agent Interactions

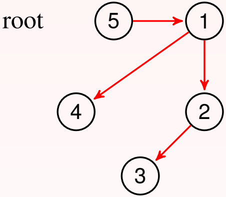
Graph: $(\mathcal{V}, \mathcal{E})$
Nodes: $\mathcal{V} = \{1, \ldots, n\}$
Edges: $\mathcal{E} = \{(i, j)\}$
i is a neighbor of j if $(i, j) \in \mathcal{E}$

A directed path
$(i_1, i_2), (i_3, i_4), \ldots$

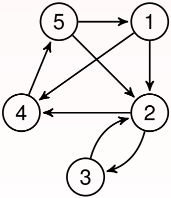




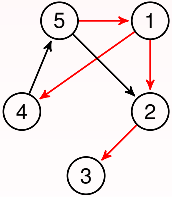
An undirected graph that is connected



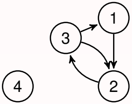
A directed spanning tree



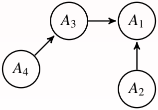
A directed spanning that is strongly connected



A graph that contains a directed spanning tree but not strongly connected



(i) Separated groups



(ii) Multiple leaders

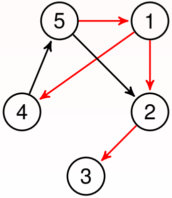
Modeling of Agent Interactions (cont.)

Adjacency Matrix

Let $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ be the adjacency matrix associated with $\mathcal{G}$, where $a_{ij} > 0$ if $(j,i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise.

(Nonsymmetric) Laplacian Matrix

Let $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ be the nonsymmetric Laplacian matrix associated with $\mathcal{G}$, where $l_{ij} = a_{ij}$, $i \neq j$, $l_{ij} = \sum_{j=1, j\neq i}^{n} a_{ij}$.



$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

adjacency matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Laplacian matrix

Some Research Directions in Multi-agent Systems

Objective

- Leaderless Consensus
- Coordinated Tracking with One Leader
- Containment Control with Multiple Leaders
- Formation Control/Flocking
- Coverage control
- Estimation...

Model

- Single Integrators
- Double Integrators
- General Linear Systems
- Nonlinear Systems
  1. Euler-Lagrange Systems
  2. Attitude Dynamics of Rigid Bodies
  3. Nonholonomic Unicycles
  4. General Nonlinear Systems

Issues

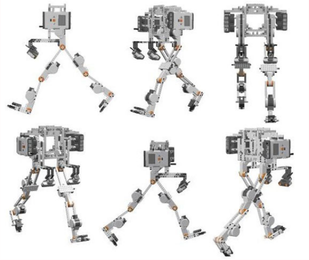
delay, switching/random network, saturation, quantized, sampled-data, finite-time, output feedback, optimization, gossip, game theory, event-triggering based,...

Coordination of Multiple Lagrangian Systems

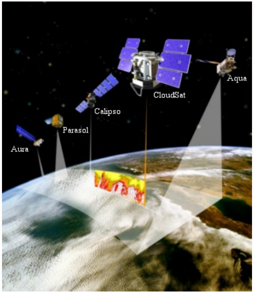
Why Lagrangian systems?

- A class of mechanical systems are Lagrangian systems and coordination of them has many applications.
- Results for single- and double-integrator dynamics cannot be directly applied due to inherent nonlinearity and parametric uncertainties in Lagrangian systems.
- Examples: robot manipulators in joint space with unknown but constant masses, inertias, and distances of the CoM of links, attitude dynamics of rigid bodies with unknown but constant inertias, and car-like robots with unknown masses and damping constants ...

L6AC-KT: http://www.hyfun.com.hk



'Pinocchio' Built With LEGOs:
http://youbentmywookie.com/wtf



A-train(NASA):
http://c3vp.org/links/links.html

Euler-Lagrange Equations

A dynamical system with p degrees of freedom can be described by the EL equations as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \quad i = 1, \cdots, n \tag{1}$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive definite inertia matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^p$ is the vector of Coriolis and centrifugal forces, $g_i(q_i)$ is the vector of gravitational force, and $\tau_i \in \mathbb{R}^p$ is the vector of control force on the ith agent.

Properties:
1) $M_i(q_i)$ is positive definite; $\|C_i(x, y)z\| \leq k_C\|y\|\|z\|$.
2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.
3) $M_i(q_i)x + C_i(q_i, \dot{q}_i)y + g_i(q_i) = Y_i(q_i, \dot{q}_i, y, x)\Theta_i$, where $Y_i(q_i, \dot{q}_i, y, x)$ is the regressor and Θ_i is an unknown but constant vector.

Distributed Coordinated Tracking with a Dynamic Leader for Multiple Euler-Lagrange Systems

Literature review

SunZhaoFeng07-TCST, SpongChopra07, ChungSlotine09-TR, CheahHouSlotine09-Automatica,...

Issues: Leader's information available to all followers.

Objective

Tracking a dynamic leader where the leader is a neighbor of only a subset of the followers and when the leader has a time-varying velocity.

Reference: J. Mei, W. Ren, G. Ma. "Distributed Coordinated Tracking with a Dynamic Leader for Multiple Euler-Lagrange Systems". IEEE Trans. on Automatic Control. 2011, 56(6): 1415-1421.

Note

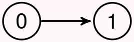
Only a fixed topology was studied here. For switching topologies, see H. Cai and J. Huang, Leader-following consensus of multiple uncertain Euler-Lagrange systems under switching network topology, IJGS, 2014 (invited).

Coordinated Tracking with a Dynamic Leader: Issues Involved

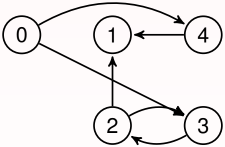
A group of followers tracks a dynamic leader when the leader is a neighbor of only a subset of the followers and all followers have only local interaction.

Leader: Agent 0 with (varying) state ξ_0 (indep. of followers)

Follower: Agent 1 to n; Follower dynamics: $\dot{\xi}_i = u_i$.



$$u_1 = \dot{\xi}_0 - (\xi_1 - \xi_0)$$



u_j $\Longrightarrow$ algebraic loop! Internal model principle explanation: [WielandSepulchreAllgower11]

$$u_i = \frac{\sum_{j \in \mathcal{N}_i} \dot{\xi}_j + b_j \dot{\xi}_0}{|\mathcal{N}_i| + b_i} - \left(\xi_i - \frac{\sum_{j \in \mathcal{N}_i} \xi_j + b_j \xi_0}{|\mathcal{N}_i| + b_i} \right)$$

$\mathcal{N}_i$: set of follower neighbors

$b_i = 1$ or 0: leader is a neighbor (yes/no).

$u_i = b_i \dot{\xi}_0 - \sum_{j \in \mathcal{N}_i} (\xi_i - \xi_j) - b_i(\xi_i - \xi_0)$ Still doesn't work!

Our goal: achieve anonym for each agent in algorithm design

Coordinated Tracking: Single-integrator Dynamics

Assumption

$$\|\dot{\xi}_0(t)\|_\infty \leq \gamma_l$$

.

Algorithm for Followers

$$u_i = -\beta \mathrm{sgn}\left[\sum_{j\in\mathcal{N}_i(t)} (\xi_i - \xi_j)\right], \quad i = 1, \ldots, n,$$

where $\beta > 0$ and $\mathrm{sgn}(\cdot)$ is the signum function (componentwise).

Convergence Result [CaoRen12]

Suppose that the follower graph is undirected and the leader has directed paths to each follower at each time instant. If $\beta > \gamma_l$, then all $\xi_i(t)$ approach $\xi_0(t)$ in finite time.

Coordinated Tracking for Multiple Lagrangian Systems: Main Result

Auxiliary Variables

$$s_i = \dot{q}_i + \lambda q_i, i = 1, \ldots, n.$$

Control Algorithm

$$\begin{aligned}\tau_i &= -\beta\Big(\sum_{j=1}^{n} a_{ij}\Big\{\mathrm{sgn}\Big[\sum_{k=0}^{n} a_{ik}(s_i - s_k)\Big] - \mathrm{sgn}\Big[\sum_{k=0}^{n} a_{jk}(s_j - s_k)\Big]\Big\} \\ &\quad + a_{i0}\mathrm{sgn}\Big[\sum_{j=0}^{n} a_{ij}(s_i - s_j)\Big]\Big). \qquad (2)\end{aligned}$$

Idea: drive $s_i \to s_0$ in finite time $=>$ $q_i \to q_0$.

Case 2: Main Result Cont.

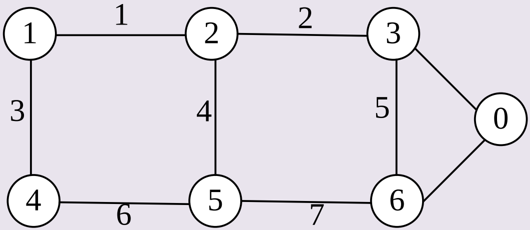
Theorem 1.2

Suppose that the follower graph is undirected and the leader has directed paths to all followers, using (2) for (1), $q_i(t) - q_0(t) \to \mathbf{0}_p$ *and* $\dot{q}_i(t) - \dot{q}_0(t) \to \mathbf{0}_p$ *exponentially as* $t \to \infty$ *if* β *is chosen large enough (a lower bound has been given).*

Simulation Result

We consider six networked two-link revolute joint arms modeled by Euler-Lagrange equations. Below is the networked topology associated with the six followers and the leader. There are seven edges between the followers, and arms 3 and 6 have access to the leader (i.e., arm 0).

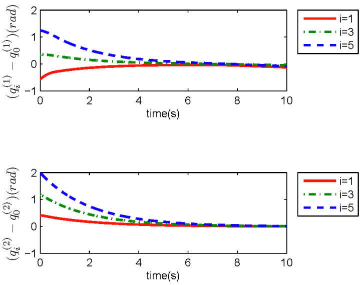
The interaction among the six followers and the leader



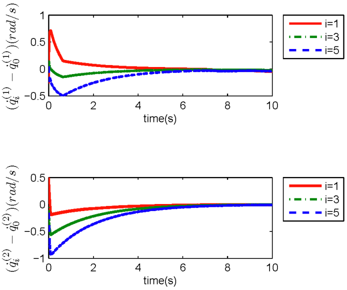
Simulation Result: Leader with a Varying Velocity

Control Algorithm (2): $q_0(t) = [\cos(\frac{2\pi}{60}t), \sin(\frac{2\pi}{60}t)]^T$rad, $\alpha = 5$, $\lambda = 0.5$, and $\beta = 8.5$. Play video: Lagrange-coor-tracking-vary-vel.avi

Errors of the joint angles



Errors of the joint angle derivatives



Containment Control with Multiple Leaders

Objective

A group of followers is driven by a group of leaders to be in the region formed by the leaders with only local interaction.

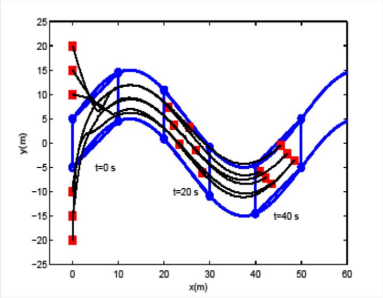
Applications

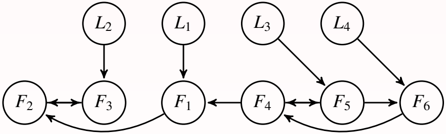
cooperative herding, hazardous material handling, and cooperative transport

Challenge

The followers do not know where the convex hull is but can only interact with local neighbors.

Leader region: changing shape, moving





Play video: a-containment-dyn-leader-fixed-top.avi

Distributed Containment Control for Multiple Lagrangian Systems with Parametric Uncertainties in Directed Networks

Literature review

JiFerraiEgerstedtBuffa08-TAC, CaoRen09-CDC, CaoStuartRenMeng11-TCST,ShiHongJohansson12-TAC,LouHong12-Automatica, DimarogonasTsiotrasKyriakopoulos09-SCL, MengRenYou10-Automatica,...

Objective

Drive a team of followers modeled by Euler-Lagrange equations to the convex hull spanned by multiple leaders under three cases:

- The leaders are stationary (leaderless consensus as a special case);
- The leaders have constant velocities;
- The leaders have varying velocities.

References:
J. Mei, W. Ren, G. Ma. Distributed Containment Control for Lagrangian Networks with Parametric Uncertainties under a Directed Graph. Automatica. 2012, 48(4): 653-659.
J. Mei, W. Ren, J. Chen, G. Ma. Distributed Adaptive Coordination for Multiple Lagrangian Systems under a Directed Graph without Using Neighbors' Velocity Information. Automatica. 2013, 49(6): 1723-1731.

Preliminary: Modeling of Interactions

Followers: agents or followers 1 to $m \longrightarrow \mathcal{V}_F$
Leaders: agents or leaders $m+1$ to $n \longrightarrow \mathcal{V}_L$
Note that the (nonsymmetric) Laplacian matrix $\mathcal{L}_A$ associated with the graph characterizing the interaction among the leaders and followers can be written as

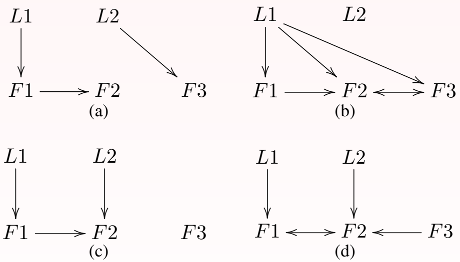
$$\mathcal{L}_A = \begin{bmatrix} L_1 & L_2 \\ \mathbf{0}_{(n-m)\times m} & \mathbf{0}_{(n-m)\times(n-m)} \end{bmatrix}, \qquad (3)$$

where $L_1 \in \mathbb{R}^{m\times m}$ and $L_2 \in \mathbb{R}^{m\times(n-m)}$.

Preliminary: Modeling of Interactions Cont.

Assumption 1

For each of the m followers, there exists at least one leader that has a directed path to the follower.



(a) and (b): Assumption 1 satisfied

(c) and (d): Assumption 1 not satisfied

Convex hull: boundary included

Lemma 1

The matrix L_1 defined as in (3) is a nonsingular M-matrix if and only if Assumption 1 holds. In addition, if Assumption 1 holds, then each entry of $-L_1^{-1}L_2$ is nonnegative and all row sums of $-L_1^{-1}L_2$ equal to one.

Containment Control: Single-integrator Dynamics

Agent Dynamics

$$\dot{\xi}_i = u_i, \quad i \in \mathcal{V}_L \bigcup \mathcal{V}_F.$$

Algorithm

$$u_i = v_i, \qquad i \in \mathcal{V}_L,$$

$\alpha > 0$

$$u_i = -\alpha \sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} (\xi_i - \xi_j) - \beta \text{sgn} \left[\sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} (\xi_i - \xi_j) \right], i \in \mathcal{V}_F,$$

where $v_i(t)$ denotes the varying velocity of leader i (indep. of followers), $\alpha > 0$, and $\beta \geq 0$.

Convergence Result [CaoRen09]:

Under Assumption 1, if $\beta \geq \gamma_l$, where $\gamma_l \triangleq \sup_{i \in \mathcal{V}_L} \|v_i(t)\|_\infty$, all followers will always converge to the dynamic convex hull spanned by the leaders.

Case 1: Stationary Leaders

Auxiliary Variables

$$\dot{q}_{ri} \triangleq -\alpha \sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} a_{ij}(q_i - q_j), \tag{4}$$

$$s_i \triangleq \dot{q}_i - \dot{q}_{ri} = \dot{q}_i + \alpha \sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} a_{ij}(q_i - q_j), \quad i \in \mathcal{V}_F, \tag{5}$$

Note (5) can be written as a vector form as

$$\dot{\bar{q}}_F = -\alpha(L_1 \otimes I_p)\bar{q}_F + s_F, \tag{6}$$

where $\bar{q}_F \triangleq q_F + (L_1^{-1} L_2 \otimes I_p) q_L$.

Idea: drive s_i to zero first, then $s_i \to \mathbf{0}_p \Longrightarrow \bar{q}_F \to \mathbf{0}_{np}$ (ISS stability and all eigenvalues of L_1 have positive real parts).

Case 1: Stationary Leaders

Control Algorithm

$$\tau_i = -K_i s_i + Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})\widehat{\Theta}_i, \tag{7a}$$

$$\dot{\widehat{\Theta}}_i = -\Lambda_i Y_i^T(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) s_i, \quad i \in \mathcal{V}_F, \tag{7b}$$

Case 1: Stationary Leaders–Main Result

Theorem 2.1

Suppose that all leaders are stationary. Using (7) for (1), $d[q_i(t), Co(q_L)] \to 0$ and $\dot{q}_i \to \mathbf{0}_p$ as $t \to \infty$, $\forall i \in \mathcal{V}_F$, for arbitrary initial conditions in the presence of parametric uncertainties if and only if Assumption 1 holds. More specifically, $q_F(t) \to -(L_1^{-1}L_2 \otimes I_p)q_L$ as $t \to \infty$, that is, the final vectors of generalized coordinates of the followers are given by $-(L_1^{-1}L_2 \otimes I_p)q_L$.

Proof Hint: Consider the following Lyapunov candidate

$$V(t) = \frac{1}{2}s_F^T M(q_F)s_F + \frac{1}{2}\widetilde{\Theta}^T \Lambda^{-1}\widetilde{\Theta} \tag{8}$$

where $\Lambda^{-1} \triangleq \text{diag}(\Lambda_1^{-1}, \cdots, \Lambda_n^{-1})$ is symmetric positive definite.

Leaderless Consensus–Main Result

Theorem 2.2

Suppose that $\mathcal{V}_L = \emptyset$.[a] Using (7) for (1), $\|q_i(t) - q_j(t)\| \to 0$ and $\dot{q}_i(t) \to \mathbf{0}_p$ as $t \to \infty$ for arbitrary initial conditions in the presence of parametric uncertainties if and only if the directed graph $\mathcal{G}$ associated with the n agents has a directed spanning tree.

[a] In this case, there does not exist a leader. Therefore, (7) becomes a leaderless consensus algorithm accounting for parametric uncertainties.

Case 1: Without Relative Neighbors' Velocities

Control Algorithm

$$\tau_i = -K_i s_i + Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \mathbf{0}_p)\widehat{\Theta}_i, \tag{9a}$$

$$\dot{\widehat{\Theta}}_i = -\Lambda_i Y_i^T(q_i, \dot{q}_i, \dot{q}_{ri}, \mathbf{0}_p)s_i, \quad i \in \mathcal{V}_F, \tag{9b}$$

Case 1: Without Relative Neighbors' Velocity–Main Result

Theorem 2.3

Suppose that all leaders are stationary. Using (9) for (1), if the control gains are chosen properly, $d[q_i(t), Co(q_L)] \to 0$ *and* $\dot{q}_i \to \mathbf{0}_p$ *as* $t \to \infty$, $\forall i \in \mathcal{V}_F$, *for arbitrary initial conditions in the presence of parametric uncertainties if and only if Assumption 1 holds. Specifically,* $q_F(t) \to -(L_1^{-1}L_2 \otimes I_p)q_L$ *as* $t \to \infty$.

Proof Hint: Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}s_F^T M(q_F)s_F + \frac{1}{2}\widetilde{\Theta}^T \Lambda^{-1}\widetilde{\Theta} + \bar{q}_F^T (D \otimes I_p)\bar{q}_F, \tag{10}$$

where Λ^{-1} is the block diagonal matrix of Λ_i^{-1}, $\forall i \in \mathcal{V}_F$ and $D \triangleq \text{diag}(d_1, \cdots, d_m)$ is a diagonal matrix with $d_i > 0$, $\forall i = 1, \ldots, m$, such that $DL_1 + L_1^T D$ is symmetric positive definite.

Leaderless Consensus: Without Relative Neighbors' Velocity–Main Result

Theorem 2.4

Suppose that $\mathcal{V}_L = \emptyset$.[a] *Using (9) for (1), choosing the control gains K_i, $i = 1, \ldots, n$, properly, $\|q_i(t) - q_j(t)\| \to 0$ and $\dot{q}_i(t) \to \mathbf{0}_p$ as $t \to \infty$ for arbitrary initial conditions in the presence of parametric uncertainties if and only if the directed graph $\mathcal{G}$ associated with the n agents has a directed spanning tree.*

[a] In this case, there does not exist a leader. Therefore, (9) becomes a leaderless consensus algorithm accounting for parametric uncertainties without using neighbors' velocity information.

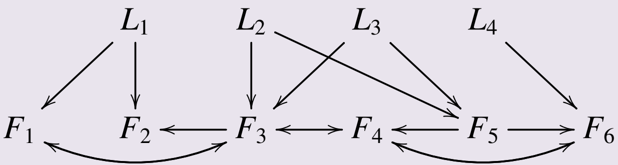
Simulation Results

We consider the containment control problem for ten agents with four leaders and six followers. The dynamic equation of each follower is modeled

$$m_i \ddot{q}_i + \beta_i \dot{q}_i = \tau_i, \quad i = 1, \dots, 6,$$

where $q_i \in \mathbb{R}^2$, and m_i and β_i represent, respectively, the mass and damping constants of the i follower, which are assumed to be constant but unknown.

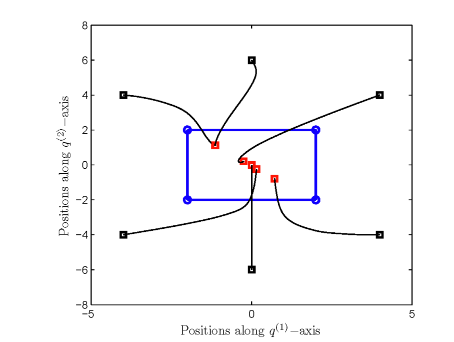
The interaction among the four leaders and the six followers



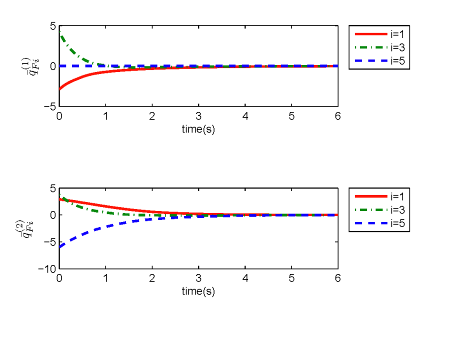
Simulation Results

Control Algorithm (7): $\alpha = 0.2$, $K_i = 0.5I_2$, $\Lambda_i = 5I_2$, $\forall i = 1,\ldots,6$.

Trajectories of the followers



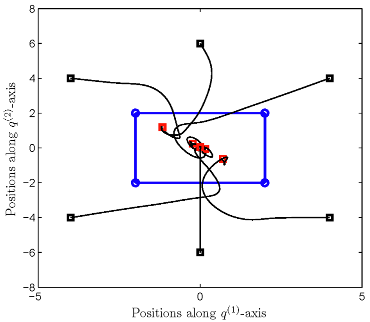
Containment error



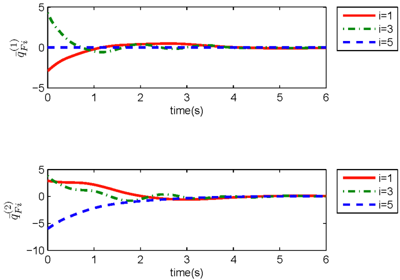
Simulation Results: Without Using Neighbors' Velocities

Control Algorithm (9): $\alpha = 0.2$, $K_i = 0.5I_2$, $\Lambda_i = 5I_2$, $\forall i = 1,\ldots,6$.

Trajectories of the followers



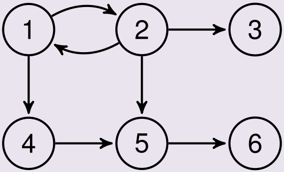
Containment error



Leaderless Consensus:: Without Using Neighbors Velocities

Synchronization of six networked robotic arms

Without using neighbors' velocities



Play video: syn-no-neighbor-vel-dir.mov

Case 2: Dynamic Leaders with Constant Velocities

Auxiliary Variables

$$\dot{q}_{ri} \triangleq \hat{v}_i - \alpha \sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} a_{ij}(q_i - q_j), \tag{11}$$

$$s_i \triangleq \dot{q}_i - \dot{q}_{ri} = \dot{q}_i - \hat{v}_i + \alpha \sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} a_{ij}(q_i - q_j), \; i \in \mathcal{V}_F, \tag{12}$$

Control Algorithm

$$\tau_i = -K_i s_i + Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})\widehat{\Theta}_i, \tag{13a}$$

$$\dot{\hat{v}}_i = -\beta\Big[\sum_{j \in \mathcal{V}_F} a_{ij}(\hat{v}_i - \hat{v}_j) + \sum_{j \in \mathcal{V}_L} a_{ij}(\hat{v}_i - \dot{q}_j)\Big], \tag{13b}$$

$$\dot{\widehat{\Theta}}_i = -\Lambda_i Y_i^T(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) s_i, \quad i \in \mathcal{V}_F, \tag{13c}$$

Define $\bar{v}_F \triangleq \hat{v}_F + (L_1^{-1} L_2 \otimes I_p)\dot{q}_L$. We have $\dot{\bar{q}}_F = -\alpha(L_1 \otimes I_p)\bar{q}_F + \bar{v}_F + s_F$.

Idea: drive s_i to zero and $\hat{v}_i$ to const first, then $s_i \to \mathbf{0}_p$ and $\bar{v}_F \to \mathbf{0}_{np} \Longrightarrow \bar{q}_F \to \mathbf{0}_{np}$.

Case 2: Dynamic Leaders with Constant Velocities–Main Result

Theorem

Suppose that the leaders have constant vectors of generalized coordinate derivatives. Using (13) for (1), $d\{q_i(t), Co[q_L(t)]\} \to 0$, $\forall i \in \mathcal{V}_F$, *as* $t \to \infty$ *for arbitrary initial conditions in the presence of parametric uncertainties if and only if Assumption 1 holds. More specifically,* $\|q_F(t) + (L_1^{-1}L_2 \otimes I_p)q_L(t)\| \to 0$ *as* $t \to \infty$.

Proof Hint: Consider the following Lyapunov candidate

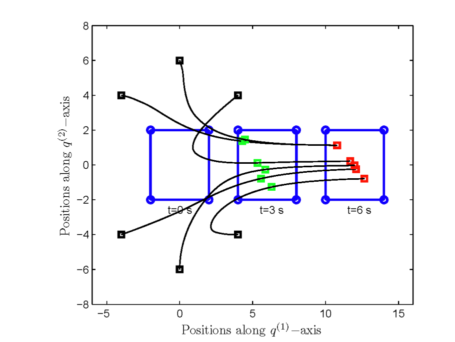
$$V(t) = \frac{1}{2}s_F^T M(q_F)s_F + \frac{1}{2}\widetilde{\Theta}^T\Lambda^{-1}\widetilde{\Theta}$$

to obtain that s_F converges to zero. Then v_F also converges to zero since $\dot{\bar{v}}_F = -\beta(L_1 \otimes I_p)\bar{v}_F$.

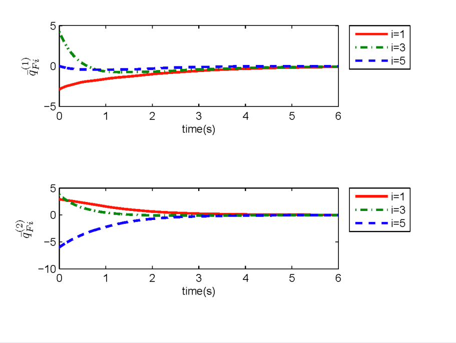
Simulation Results: Leaders with Constant Velocities

Control algorithm (13): Let the initial positions of the four leaders be, respectively, $[-2, 2]^T$, $[2, 2]^T$, $[-2, -2]^T$, and $[2, -2]^T$, and the velocities be identical, $[2, 0]^T$. $\alpha = 0.5$, $K_i = 0.8I_2$, $\Lambda_i = 5I_2$, $\forall i = 1,\ldots,6$, $\beta = 1$.

Trajectories of the followers



Containment error



Case 3: Dynamic Leaders with Varying Velocities

Auxiliary Variables

$$\hat{\dot{q}}_{ri} \triangleq \hat{v}_i - \alpha \sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} a_{ij}(q_i - q_j), \tag{14}$$

$$\hat{\ddot{q}}_{ri} \triangleq \hat{a}_i - \alpha \sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} a_{ij}(\dot{q}_i - \dot{q}_j), \tag{15}$$

$$\hat{s}_i \triangleq \dot{q}_i - \hat{\dot{q}}_{ri} = \dot{q}_i - \hat{v}_i + \alpha \sum_{j \in \mathcal{V}_L \bigcup \mathcal{V}_F} a_{ij}(q_i - q_j), \quad i \in \mathcal{V}_F, \tag{16}$$

Case 3: Dynamic Leaders with Varying Velocities

Control Algorithm

$$\tau_i = -K_i\hat{s}_i + Y_i(q_i, \dot{q}_i, \hat{\dot{q}}_{ri}, \hat{\ddot{q}}_{ri})\widehat{\Theta}_i, \tag{17a}$$

$$\dot{\hat{v}}_i = -\beta_1 \text{sgn}\Big[\sum_{j\in\mathcal{V}_F} a_{ij}(\hat{v}_i - \hat{v}_j) + \sum_{j\in\mathcal{V}_L} a_{ij}(\hat{v}_i - \dot{q}_j)\Big] \tag{17b}$$

$$\dot{\hat{a}}_i = -\beta_2 \text{sgn}\Big[\sum_{j\in\mathcal{V}_F} a_{ij}(\hat{a}_i - \hat{a}_j) + \sum_{j\in\mathcal{V}_L} a_{ij}(\hat{a}_i - \ddot{q}_j)\Big], \tag{17c}$$

$$\dot{\widehat{\Theta}}_i = -\Lambda_i Y_i^T(q_i, \dot{q}_i, \hat{\dot{q}}_{ri}, \hat{\ddot{q}}_{ri})\hat{s}_i, \quad i\in\mathcal{V}_F, \tag{17d}$$

Let $q_d \triangleq [q_{d1}^T, \ldots, q_{dm}^T]^T = -(L_1^{-1}L_2 \otimes I_p)q_L$, where $q_{di}\in\mathbb{R}^p$. Define $s_i = \dot{q}_i - \dot{q}_{di} + \alpha \sum_{j\in\mathcal{V}_L\bigcup\mathcal{V}_F} a_{ij}(q_i - q_j)$.

Idea: drive $\hat{v}_i$ to $\dot{q}_{di}$ and $\hat{a}_i$ to $\ddot{q}_{di}$ in finite time; then drive $\hat{s}_i(s_i)$ to zero; and last $s_i \to \mathbf{0}_{np}$ and $\bar{v}_F \to \mathbf{0}_{np} \Longrightarrow \bar{q}_F \to \mathbf{0}_{np}$.

Case 3: Dynamic Leaders with Varying Velocities–Main Result

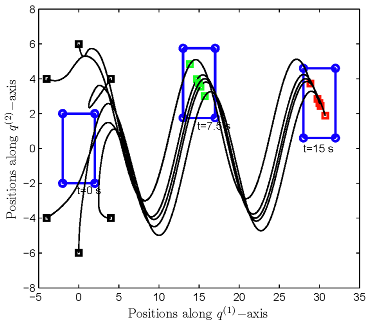
Theorem

Suppose that the leaders have varying vectors of generalized coordinate derivatives, $\beta_1 > \|\ddot{q}_d\|$, and $\beta_2 > \|\dddot{q}_d\|$, where $q_d \triangleq -(L_1^{-1}L_2 \otimes I_p)q_L$. Using (17) for (1), $d\{q_i(t), Co[q_L(t)]\} \to 0$ as $t \to \infty$, $\forall i \in \mathcal{V}_F$, for arbitrary initial conditions in the presence of parametric uncertainties if and only if Assumption 1 holds. More specifically, $\|q_F(t) + (L_1^{-1}L_2 \otimes I_p)q_L(t)\| \to 0$ as $t \to \infty$.

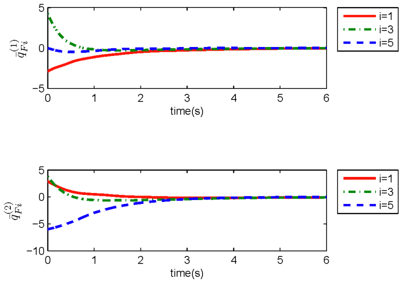
Simulation results: Leaders with Varying Velocities

Control Algorithm (17): Let the initial positions of the fours leaders be, respectively, $[-2,2]^T$, $[2,2]^T$, $[-2,-2]^T$, and $[2,-2]^T$, the initial velocities be identical, $[2,4]^T$, and the accelerations be identical, $[0,-4\sin(t)]^T$. $\alpha = 0.5$, $K_i = 0.8I_2$, $\Lambda_i = 5I_2$, $\forall i = 1,\ldots,6$, $\beta_1 = \beta_2 = 4$.

Trajectories of the followers



Containment error



Distributed Containment Control for Multiple Lagrangian Systems in the Presence of Unknown Uncertainties and External Disturbances

Agent Dynamics

The m followers are represented by the following Lagrangian equations

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i + \omega_i, \quad i = 1, \cdots, m \tag{18}$$

where ω_i is the external disturbance and M_i, C_i, and g_i are unknown.

Literature review

HouChengTan09-SMCB (Limitation: undirected graph)
DasLewis10-Automatica, DasLewis11-IJRNC, ChenLewis11-SMCB, ZhangLewis12-Automatica, ZhangLewisQu12-TIE (Limitation: Both Laplacian matrix and pinning gains (global information) needed for neural network updating laws)

Objective

Drive a team of followers modeled by unknown Euler-Lagrange equations to the convex hull spanned by multiple dynamics leaders under two cases:

- Using both relative position and velocity feedback;
- Without using relative velocity feedback.

Reference:
J. Mei, W. Ren, B. Li, G. Ma. Containment Control for Networked Unknown Lagrangian Systems with Multiple Dynamic Leaders under a Directed Graph. ACC. 2013, accepted.

Case 1: Using both relative position and velocity feedback-Dynamics

Auxiliary Variables

$$\dot{q}_{ri} \triangleq -\alpha \sum_{j\in\mathcal{V}_L\bigcup\mathcal{V}_F} a_{ij}(q_i - q_j),$$

$$s_i \triangleq \dot{q}_i - \dot{q}_{ri} = \dot{q}_i + \alpha \sum_{j\in\mathcal{V}_L\bigcup\mathcal{V}_F} a_{ij}(q_i - q_j),\ i \in \mathcal{V}_F,$$

Then (18) can be written as

$$M_i(q_i)\dot{s}_i + C_i(q_i,\dot{q}_i)s_i = f_i(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri}) + \omega_i + \tau_i, \tag{19}$$

where $f_i(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri}) \triangleq -M_i(q_i)\ddot{q}_{ri} - C_i(q_i,\dot{q}_i)\dot{q}_{ri} - g_i(q_i)$ is unknown since $M_i(q_i)$, $C_i(q_i,\dot{q}_i)$, and $g_i(q_i)$ are all unknown.

Case 1: Using both relative position and velocity feedback-Approximation

Due to the approximation property of neural networks, the unknown nonlinearity $f_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})$ can be approximated as

$$f_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) = W_i^T \phi_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) + \varepsilon_i,$$

W_i: the ideal constant approximating weight matrix;
$\phi_i(\cdot)$: a suitable basis set of functions;
ε_i: the approximation error (assumed bounded over a compact set).

The estimate of $f_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})$ can be written as

$$\hat{f}_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) = \widehat{W}_i^T \phi_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}), \quad i \in \mathcal{V}_F, \tag{20}$$

where $\widehat{W}_i$ is the estimate of W_i to be designed later.

Case 1: Using both relative position and velocity feedback-Assumptions

Besides Assumption 1 on the interaction topology among the agents, the following two assumptions are made.

Assumption 2

All leaders' states and state derivatives, q_i, $\dot{q}_i$, $i \in \mathcal{V}_L$, are bounded.

Assumption 3

The external disturbance ω_i is bounded.

Case 1: Using both relative position and velocity feedback–Control Algorithm

Control Algorithm with Unknown Nonlinearities and External Disturbances

$$\tau_i = -K_i s_i - \widehat{W}_i^T \phi_i - \hat{k}_i \mathrm{sgn}(s_i), \tag{21a}$$

$$\dot{\widehat{W}}_i = \gamma_i \phi_i s_i^T, \tag{21b}$$

$$\dot{\hat{k}}_i = \delta_i \|s_i\|_1, \qquad i \in \mathcal{V}_F, \tag{21c}$$

where K_i is a symmetric positive-definite matrix, γ_i and δ_i are positive constants.

Idea: drive s_i to zero first , then $s_i \to \mathbf{0}_p$ and $\dot{q}_L$ being bounded $\Longrightarrow$ $\bar{q}_F$ can be reduced as small as desired.

Case 1: Using both relative position and velocity feedback–Main Result

Theorem

Under Assumption 2 and Assumption 3, using (21) *for* (18)*, the containment error* $\bar{q}_F$ *can be reduced as small as desired by tuning* α *for arbitrary initial conditions in a compact set if and only if Assumption 1 holds. More specially, with an additional assumption that* $\lim_{t\to\infty} \|\dot{q}_L(t)\| = 0$*, the containment error will converge to zero asymptotically.*

Proof Hint: First Step–drive s_i to zero: Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2} s_F^T M(q_F) s_F + \sum_{i\in\mathcal{V}_F} \frac{1}{2\gamma_i} \mathrm{tr}(\widetilde{W}_i^T \widetilde{W}_i) + \sum_{i\in\mathcal{V}_F} \frac{1}{2\delta_i} (k_i - \hat{k}_i)^2. \tag{22}$$

Second Step–reduce $\bar{q}_F$ as small as desired: Consider the Lyapunov function candidate

$$V_1(t) = \bar{q}_F^T (D \otimes I_p) \bar{q}_F \tag{23}$$

with $D \triangleq \mathrm{diag}(d_1, \cdots, d_m)$ being a positive diagonal matrix such that $Q \triangleq L_1^T D + DL_1$ is symmetric positive definite.

Case 2: Without using relative velocity feedback

When relative velocity measurements are not available in the absence of communication, (19) can be rewritten as

$$M_i(q_i)\dot{s}_i + C_i(q_i,\dot{q}_i)s_i = -M_i(q_i)\ddot{q}_{ri} + f_i(q_i,\dot{q}_i,\dot{q}_{ri}) + \omega_i + \tau_i, \tag{24}$$

where $f_i(q_i,\dot{q}_i,\dot{q}_{ri}) \triangleq -C_i(q_i,\dot{q}_i)\dot{q}_{ri} - g_i(q_i)$.

The unknown nonlinearity $f_i(q_i,\dot{q}_i,\dot{q}_{ri})$ can be approximated as $f_i(q_i,\dot{q}_i,\dot{q}_{ri}) = W_i^T\phi_i(q_i,\dot{q}_i,\dot{q}_{ri}) + \varepsilon_i$ and the estimate of $f_i(q_i,\dot{q}_i,\dot{q}_{ri},)$ can be written as $\hat{f}_i(q_i,\dot{q}_i,\dot{q}_{ri}) = \widehat{W}_i^T\phi_i(q_i,\dot{q}_i,\dot{q}_{ri}), \forall i \in \mathcal{V}_F$.

Control Algorithm

$$\tau_i = -\hat{h}_i s_i - \widehat{W}_i^T\phi_i - \hat{l}_i \mathrm{sgn}(s_i), \tag{25a}$$

$$\dot{\hat{h}}_i = \gamma(s_i^T s_i - \nu\hat{h}_i), \tag{25b}$$

$$\dot{\widehat{W}}_i = \gamma(\phi_i s_i^T - \nu\widehat{W}_i), \tag{25c}$$

$$\dot{\hat{l}}_i = \gamma(\|s_i\|_1 - \nu\hat{l}_i), \qquad i \in \mathcal{V}_F, \tag{25d}$$

where γ and ν are positive constants.

Case 2: Without using relative velocity feedback–Main Result

Theorem

Under Assumption 2 and Assumption 3, using (25) *for* (18)*, the containment error can be reduced as small as desired by tuning* α *for arbitrary initial conditions in a compact set if and only if Assumption 1 holds.*

Proof Hint: First Step–show the boundedness of $s_F(t)$ and $\bar{q}_F(t)$: Consider the following Lyapunov function candidate

$$V_c(t) = \frac{1}{2} s_F^T M(q_F) s_F + \bar{q}_F^T (D \otimes I_p) \bar{q}_F + \frac{1}{2\gamma} \sum_{i \in \mathcal{V}_F} \left[\operatorname{tr}(\widetilde{W}_i^T \widetilde{W}_i) + (h_c - \hat{h}_i)^2 + (l_c - \hat{l}_i)^2 \right], \tag{26}$$

where l_c is chosen such that $\|r_i + \omega_i + \varepsilon_i\| \leq l_c$ with $r \triangleq \alpha M(q_F)(L_2 \otimes I_p)\dot{q}_L = [r_1^T, \ldots, r_m^T]^T$, and h_c is a positive constant to be chosen large enough.

Second Step–reduce $\bar{q}_F$ as small as desired: Consider the following Lyapunov function candidate

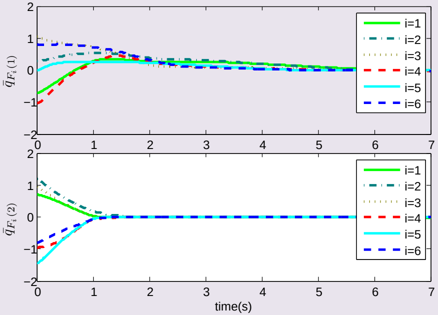
$$V(t) = \frac{1}{2} s_F^T M(q_F) s_F + \frac{1}{2\gamma} \sum_{i \in \mathcal{V}_F} \operatorname{tr}(\widetilde{W}_i^T \widetilde{W}_i) + \frac{1}{2\gamma} \sum_{i \in \mathcal{V}_F} \left[(h - \hat{h}_i)^2 + (l - \hat{l}_i)^2 \right], \tag{27}$$

where l is chosen such that $\|M_i(q_i)\ddot{q}_{ri} + \omega_i + \varepsilon_i\| \leq l, \forall i \in \mathcal{V}_F$, and h is a positive constant.

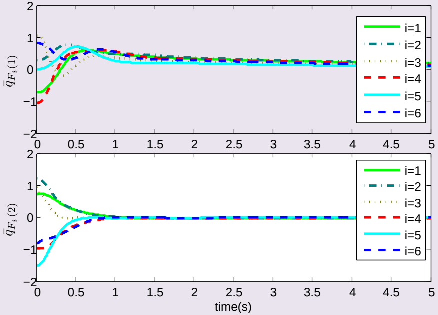
Simulation results

Below are the containment errors using, respectively, control algorithms (21) and (25) for a team with six followers and four leaders.

Containment Error using (21)



Containment error using (25)



Conclusion

Conclusion

- Coordinated tracking with a single leader
- Containment control with multiple stationary/dynamic leaders in the presence of parametric uncertainties
- Containment control with multiple dynamic leaders in the presence of unknown uncertainties and external disturbances

Thank You!

Thank You!
Any Question?