Applied Mathematics and Computation 221 (2013) 111–120

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stability analysis of static recurrent neural networks with interval time-varying delay

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ARTICLE INFO

Keywords:
Delay-dependent stability
Interval time-varying delay
Static recurrent neural network
Linear matrix inequality

ABSTRACT

The problem of stability analysis of static recurrent neural networks with interval time-varying delay is investigated in this paper. A new Lyapunov functional which contains some new double integral and triple integral terms are introduced. Information about the lower bound of the delay is fully used in the Lyapunov functional. Integral and double integral terms in the derivative of the Lyapunov functional are divided into some parts to get less conservative results. Some sufficient stability conditions are obtained in terms of linear matrix inequality (LMI). Numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

During past several decades, recurrent neural networks have been applied in many areas such as speech recognition, handwriting recognition, optimization problem, model identification and automatic control [1,2]. Although neural networks can be implemented by very large scale integrated circuits, there inevitably exist some delays in neural networks due to the limitation of the speed of transmission and switching of signals. It is well known that time-delay is usually a cause of instability and oscillations of recurrent neural networks. Therefore, the problem of stability of recurrent neural networks with time-delay is of importance in both theory and practice. Many results on this topic have been obtained which can be classified into delay-dependent ones and delay-independent ones. Since delay-dependent stability conditions are usually less conservative than delay-independent ones, much attention has been put into developing some less conservative delay-dependent stability conditions [3–24].

Neural networks can be classified into two categories, that is, static neural networks and local field networks. In static neural networks, neuron states are chosen as basic variables. While in local field networks, local field states are chosen as basic variables. It has been proved that these two kinds of neural networks are not always equivalent [25]. Compared with rich results for local field networks, results for static neural networks are much more scare. To mention a few, stability of static recurrent neural networks with constant time-delay was investigated in [26] where new delay-dependent stability criteria were established in the terms of LMI using delay-partitioning approach and Finsler's lemma. By introducing some slack matrices, delay-dependent stability conditions for static recurrent neural networks with time-varying delay were obtained and expressed as LMIs [27]. By constructing a new Lyapunov functional and using s-procedure, both delay-dependent and delay-independent stability conditions were developed for static recurrent neural networks with interval time-varying delays in [28]. Stability and dissipativity analysis of static neural networks were investigated in [29].

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http://dx.doi.org/10.1016/j.amc.2013.06.028

In this paper, stability analysis problem of static recurrent neural networks with interval time-varying delays is investigated. As mentioned in our previous works [32,33], information about the lower bound of time-varying delay should be taken into account when constructing a Lyapunov functional. Therefore, a new Lyapunov functional containing some new double-integral terms and triple-integral terms is introduced. Information about the lower bound of delay is more sufficiently used in the Lyapunov functional. Based on the new Lyapunov functional, some less conservative delay-dependent stability conditions are derived. Numerical examples are given to confirm the effectiveness of the proposed method.

Notations. Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes an n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices; $P>0$ means that the matrix P is symmetric positive definite; I is an appropriately dimensional identity matrix.

2. Problem formulation

Consider the following static recurrent neural network with interval time-varying delay:

$$\dot{u}(t) = -Au(t) + g(Wu(t-d(t)) + J), \tag{1}$$

where $u(\cdot) = [u_1(\cdot)u_2(\cdot)\cdots u_n(\cdot)]^T$ is the neuron state vector, $A = \mathrm{diag}\{a_1, a_2, \cdots, a_n\}$ with $a_i > 0, i = 1, 2, \cdots, n$, $g(Wu(\cdot)) = [g_1(W_1u(\cdot))g_2(W_2u(\cdot))\cdots g_n(W_nu(\cdot))]^T$ is the neuron activation function. $W = [W_1^TW_2^T\cdots W_n^T]^T$ is the delayed connection weight matrix. $J = [j_1, j_2, \cdots, j_n]^T$ is a constant input. $d(t)$ is the time-varying delay and satisfies

$$h_1 \leqslant d(t) \leqslant h_2 \tag{2}$$

and

$$\dot{d}(t) \leqslant \mu, \tag{3}$$

where $0 < h_1 < h_2$ and μ are constants.

The following assumption is made in this paper.

Assumption 1. Each bounded neuron activation function, $g_i(\cdot), i = 1, 2, \cdots, n$ satisfies

$$b_i \leqslant \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leqslant l_i, \quad \forall s_1, s_2 \in \mathbb{R}, s_1 \neq s_2, \quad i = 1, 2, \cdots, n, \tag{4}$$

where $b_i, l_i, i = 1, 2, \cdots, n$ are known real constants.

Assumption 1 guarantees the existence of an equilibrium point of system (1) [30,31]. Denote that $u^* = [u_1^*u_2^*\cdots u_n^*]$ is the equilibrium point. Using the transformation $x(\cdot) = u(\cdot) - u^*$, system (1) can be converted to the following error system:

$$\dot{x}(t) = -Ax(t) + f(Wx(t-d(t))), \tag{5}$$

where $x(\cdot) = [x_1(\cdot)x_2(\cdot)\cdots x_n(\cdot)]^T$ is the state vector, $f(Wx(\cdot)) = [f_1(W_1x(\cdot))f_2(W_2x(\cdot))\cdots f_n(W_nx(\cdot))]^T$ with $f(Wx(\cdot)) = g(W(x(\cdot)+u^*)+J) - g(Wu^*+J)$. It is easy to see that $f_i(\cdot), i = 1, 2, \cdots, n$, satisfies

$$b_i \leqslant \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leqslant l_i, \quad f_i(0) = 0, \quad \forall s_1, s_2 \in \mathbb{R}, s_1 \neq s_2, \quad i = 1, 2, \cdots, n. \tag{6}$$

The following integral inequalities are introduced in the following lemma which is important in the derivation of main results.

Lemma 1 ([32,34]). *For any constant matrix $Z > 0$ and scalars $0 < h_1 < h_2, h_{12} = h_2 - h_1$ such that the following integrations are well defined, then*

(1) $-\int_{t-h_2}^{t-h_1}\omega^T(s)Z\omega(s)\mathrm{d}s \leqslant -\frac{1}{h_{12}}\int_{t-h_2}^{t-h_1}\omega^T(s)\mathrm{d}sZ\int_{t-h_2}^{t-h_1}\omega(s)\mathrm{d}s,$

(2) $-\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1}\omega^T(s)Z\omega(s)\mathrm{d}s\mathrm{d}\theta \leqslant -\frac{2}{h_{12}^2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1}\omega^T(s)\mathrm{d}s\mathrm{d}\theta Z\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1}\omega(s)\mathrm{d}s\mathrm{d}\theta.$

3. Main results

In this section, some new delay-dependent stability criteria are derived by introducing a new Lyapunov functional and using a new method to estimate the derivative of the Lyapunov functional.

Denote $\hat{\theta}(t) = \mathtt{col}\{x(t), x(t-d(t)), x(t-h_1), x(t-h_2), f(Wx(t)), f(Wx(t-d(t))), \dot{x}(t-d(t)), \dot{x}(t-h_1), \dot{x}(t-h_2), \int_{t-h_1}^{t} x(s)\mathrm{d}s, f(Wx(t-h_1)), f(Wx(t-h_2))\}$. The following theorem presents a delay-dependent stability condition for system (5).

Theorem 2. *For given scalars $0 < h_1 < h_2$ and μ, system (5) is asymptotically stable for any time-varying delay satisfying (2), (3) if there exist matrices $P = [P_{ij}]_{5\times5} > 0, Q = [Q_{ij}]_{3\times3} \geqslant 0, Z = [Z_{ij}]_{3\times3} \geqslant 0, R = [R_{ij}]_{3\times3} \geqslant 0$, $X = [X_{ij}]_{2\times2} \geqslant 0, S = [S_{ij}]_{2\times2} \geqslant 0$, $U_1 \geqslant 0, U_2 \geqslant 0$, nonnegative diagonal matrices K, T and Λ and any matrices $M = \begin{bmatrix} M_1^T & M_2^T & M_3^T \end{bmatrix}^T$, $N = \begin{bmatrix} N_1^T & N_2^T & N_3^T \end{bmatrix}^T, H = \begin{bmatrix} H_1^T & H_2^T & H_3^T \end{bmatrix}^T$, $F = \begin{bmatrix} F_1^T & F_2^T & F_3^T \end{bmatrix}^T$ with appropriate dimensions such that the following LMIs holds:*

$$\begin{bmatrix} \Delta & h_{12}\Upsilon_1 & -h_{12}\hat{M} & \frac{h_{12}^2}{2}\hat{H} & A_c^T Y \\ * & -h_{12}S_{11} & -h_{12}S_{12} & 0 & 0 \\ * & * & -h_{12}S_{22} & 0 & 0 \\ * & * & * & -\frac{h_{12}^2}{2}U_2 & 0 \\ * & * & * & * & -Y \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} \Theta & h_{12}\Upsilon_2 & -h_{12}\hat{N} & \frac{h_{12}^2}{2}\hat{F} & A_c^T Y \\ * & -h_{12}S_{11} & -h_{12}S_{12} & 0 & 0 \\ * & * & -h_{12}S_{22} & 0 & 0 \\ * & * & * & -\frac{h_{12}^2}{2}U_2 & 0 \\ * & * & * & * & -Y \end{bmatrix} < 0, \tag{8}$$

where

$$\Delta = \Xi + \hat{H}E_3,$$
$$\Theta = \Xi + \hat{F}E_2 - (E_3 - E_2)U_2(E_3 - E_2)^T,$$
$$\Xi = \begin{bmatrix} [\Xi_{ij}]_{7\times7} & \Gamma \\ * & \Omega \end{bmatrix},$$

$$\begin{aligned}
\Xi_{11} &= -P_{11}A - A^T P_{11} - Z_{12}A - A^T Z_{12}^T - h_1 X_{12}A - h_1 A^T X_{12}^T + P_{14} + P_{14}^T + Z_{11} + h_1 X_{11} \\
&\quad - \frac{1}{h_1}X_{22} - 2U_1 - W^T BKLW - W^T LKBW, \\
\Xi_{12} &= -M_1 + N_1, \\
\Xi_{13} &= -A^T P_{12} + P_{24}^T + \frac{1}{h_1}X_{22} + M_1 - P_{14} + P_{15}, \\
\Xi_{14} &= -A^T P_{13} + P_{34}^T - P_{15} - N_1, \\
\Xi_{15} &= -A^T Z_{23} + Z_{13} + W^T LK + W^T BK - A^T W^T \Lambda, \\
\Xi_{16} &= P_{11} + Z_{12} + h_1 X_{12},
\end{aligned}$$

$$\begin{aligned}
\Xi_{22} &= -(1-\mu)Q_{11} - M_2 - M_2^T + N_2 + N_2^T - W^T BTLW - W^T LTBW, \\
\Xi_{23} &= M_2, \\
\Xi_{24} &= -N_2, \\
\Xi_{26} &= -(1-\mu)Q_{13} - M_3^T + N_3^T + W^T LT + W^T BT, \\
\Xi_{27} &= -(1-\mu)Q_{12}, \\
\Xi_{33} &= -P_{24} - P_{24}^T + P_{25} + P_{25}^T - Z_{11} + R_{11} + h_{12}S_{11} - \frac{1}{h_1}X_{22} + Q_{11}, \\
\Xi_{34} &= -P_{25} - P_{34}^T + P_{35}^T, \\
\Xi_{44} &= -P_{35} - P_{35}^T - R_{11}, \\
\Xi_{36} &= P_{12}^T + M_3^T, \\
\Xi_{46} &= P_{13}^T - N_3^T,
\end{aligned}$$

$$\Xi_{55} = Z_{33} - 2K,$$
$$\Xi_{56} = Z_{23}^T + \Lambda W,$$
$$\Xi_{66} = -(1-\mu)Q_{33} - 2T,$$
$$\Xi_{67} = -(1-\mu)Q_{23}^T,$$
$$\Xi_{77} = -(1-\mu)Q_{22},$$
$$\Xi_{17} = \Xi_{25} = \Xi_{35} = \Xi_{37} = \Xi_{45} = \Xi_{47} = \Xi_{57} = 0,$$
$$\Gamma = \begin{bmatrix} P_{12} & P_{13} & \Gamma_{13} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \Gamma_{31} & P_{23} & \Gamma_{33} & \Gamma_{34} & 0 \\ P_{23}^T & P_{33} - R_{12} & -P_{45}^T & 0 & -R_{13} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Gamma_{13} = -A^T P_{14} + P_{44} - \frac{1}{h_1} X_{12}^T + \frac{2}{h_1} U_1,$$
$$\Gamma_{31} = P_{22} - Z_{12} + R_{12} + h_{12} S_{12} + Q_{12},$$
$$\Gamma_{33} = -P_{44} + P_{45}^T + \frac{1}{h_1} X_{12}^T,$$
$$\Gamma_{34} = -Z_{13} + R_{13} + Q_{13},$$
$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & P_{24} & \Omega_{14} & 0 \\ * & -R_{22} & P_{34} & 0 & -R_{23} \\ * & * & -\frac{1}{h_1} X_{11} - \frac{2}{h_1^2} U_1 & 0 & 0 \\ * & * & * & -Z_{33} + R_{33} + Q_{33} & 0 \\ * & * & * & * & -R_{33} \end{bmatrix},$$
$$\Omega_{11} = -Z_{22} + R_{22} + Q_{22} + h_{12} S_{22} + \frac{h_{12}^2}{2} U_2,$$
$$\Omega_{14} = -Z_{23} + R_{23} + Q_{23},$$
$$L = \mathrm{diag}\{l_1, l_2, \cdots, l_n\},$$
$$B = \mathrm{diag}\{b_1, b_2, \cdots, b_n\},$$
$$A_c = [-A \quad 0 \quad 0 \quad 0 \quad 0 \quad I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T,$$
$$E_2 = [0 \quad I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T,$$
$$E_3 = [0 \quad 0 \quad I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T,$$
$$\hat{Y} = Z_{22} + h_1 X_{22} + h_{12} S_{22} + \frac{h_1^2}{2} U_1,$$
$$\hat{H} = \begin{bmatrix} H_1^T & H_2^T & 0 & 0 & 0 & H_3^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\hat{F} = \begin{bmatrix} F_1^T & F_2^T & 0 & 0 & 0 & F_3^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\hat{M} = \begin{bmatrix} M_1^T & M_2^T & 0 & 0 & 0 & M_3^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\hat{N} = \begin{bmatrix} N_1^T & N_2^T & 0 & 0 & 0 & N_3^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

Proof. Construct the following Lyapunov functional

$$V(x(t)) = \sum_{i=1}^{8} V_i(x(t)), \tag{9}$$

with

$$V_1(x(t)) = \zeta^T(t)P\zeta(t) + 2\sum_{i=1}^{n}\lambda_i\int_0^{W_ix(t)} f_i(s)\mathrm{d}s,$$
$$V_2(x(t)) = \int_{t-d(t)}^{t-h_1}\xi^T(s)Q\xi(s)\mathrm{d}s,$$
$$V_3(x(t)) = \int_{t-h_1}^{t}\xi^T(s)Z\xi(s)\mathrm{d}s,$$
$$V_4(x(t)) = \int_{t-h_2}^{t-h_1}\xi^T(s)R\xi(s)\mathrm{d}s,$$
$$V_5(x(t)) = \int_{-h_1}^{0}\int_{t+\theta}^{t} z^T(s)Xz(s)\mathrm{d}s\mathrm{d}\theta,$$
$$V_6(x(t)) = \int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1} z^T(s)Sz(s)\mathrm{d}s\mathrm{d}\theta,$$
$$V_7(x(t)) = \int_{-h_1}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta,$$
$$V_8(x(t)) = \int_{-h_2}^{-h_1}\int_{\theta}^{-h_1}\int_{t+\lambda}^{t-h_1}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta,$$

where $\zeta(t) = \mathtt{col}\left\{x(t), x(t-h_1), x(t-h_2), \int_{t-h_1}^{t} x(s)\mathrm{d}s, \int_{t-h_2}^{t-h_1} x(s)\mathrm{d}s\right\}, z(s) = \mathtt{col}\{x(s), \dot{x}(s)\}$ and $\xi(s) = \mathtt{col}\{x(s), \dot{x}(s), f(Wx(s))\}$.

Taking the derivative of $V(x(t))$ along the trajectories of system (5) yields

$$\dot{V}_1(x(t)) = 2\zeta^T(t)P\dot{\zeta}(t) + 2\sum_{i=1}^{n}\lambda_i W_i f_i(W_ix(t))\dot{x}_i(t) = 2\zeta^T(t)P\dot{\zeta}(t) + 2f^T(Wx(t))\Lambda W\dot{x}(t), \tag{10}$$

$$\dot{V}_2(x(t)) = \xi^T(t-h_1)Q\xi(t-h_1) - \left(1-\dot{d}(t)\right)\xi^T(t-d(t))Q\xi(t-d(t)), \tag{11}$$

$$\dot{V}_3(x(t)) = \xi^T(t)Z\xi(t) - \xi^T(t-h_1)Z\xi(t-h_1)), \tag{12}$$

$$\dot{V}_4(x(t)) = \xi^T(t-h_1)R\xi(t-h_1) - \xi^T(t-h_2)R\xi(t-h_2)), \tag{13}$$

$$\dot{V}_5(x(t)) = h_1z^T(t)Xz(t) - \int_{t-h_1}^{t} z^T(s)Xz(s)\mathrm{d}s, \tag{14}$$

$$\begin{aligned}\dot{V}_6(x(t)) &= h_{12}z^T(t-h_1)Sz(t-h_1) - \int_{t-h_2}^{t-h_1} z^T(s)Sz(s)\mathrm{d}s \\ &= h_{12}z^T(t-h_1)Sz(t-h_1) - \int_{t-d(t)}^{t-h_1} z^T(s)Sz(s)\mathrm{d}s - \int_{t-h_2}^{t-d(t)} z^T(s)Sz(s)\mathrm{d}s,\end{aligned} \tag{15}$$

$$\dot{V}_7(x(t)) = \frac{h_1^2}{2}\dot{x}^T(t)U_1\dot{x}(t) - \int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \tag{16}$$

$$\begin{aligned}\dot{V}_8(x(t)) &= \frac{h_{12}^2}{2}\dot{x}^T(t-h_1)U_2\dot{x}(t-h_1) - \int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &= \frac{h_{12}^2}{2}\dot{x}^T(t-h_1)U_2\dot{x}(t-h_1) - \int_{-d(t)}^{-h_1}\int_{t+\theta}^{t-h_1}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta - \int_{-h_2}^{-d(t)}\int_{t+\theta}^{t-d(t)}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad - (h_2 - d(t))\int_{t-d(t)}^{t-h_1}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s,\end{aligned} \tag{17}$$

Similar to [35], the following equations hold:

$$2\theta^T(t)M\left[x(t-h_1) - x(t-d(t)) - \int_{t-d(t)}^{t-h_1}\dot{x}(s)\mathrm{d}s\right] = 0, \tag{18}$$

$$2\theta^T(t)N\left[x(t-d(t)) - x(t-h_2) - \int_{t-h_2}^{t-d(t)}\dot{x}(s)\mathrm{d}s\right] = 0, \tag{19}$$

$$2\theta^T(t)H\left[(d(t)-h_1)x(t-h_1)-\int_{t-d(t)}^{t-h_1}x(s)\mathrm{d}s-\int_{-d(t)}^{-h_1}\int_{t+\theta}^{t-h_1}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right]=0, \tag{20}$$

$$2\theta^T(t)F\left[(h_2-d(t))x(t-d(t))-\int_{t-h_2}^{t-d(t)}x(s)\mathrm{d}s-\int_{-h_2}^{-d(t)}\int_{t+\theta}^{t-d(t)}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right]=0, \tag{21}$$

where $\theta(t)=\mathtt{col}\{x(t),x(t-d(t)),f(Wx(t-d(t)))\}$.

From (6), for any $K=\mathrm{diag}\{K_1,K_2,\cdots,K_n\}\geqslant 0, T=\mathrm{diag}\{T_1,T_2,\cdots,T_n\}\geqslant 0$, the following inequalities hold

$$0\leqslant 2\left[f^T(Wx(t))-x^T(t)W^TB\right]K[LWx(t)-f(Wx(t))], \tag{22}$$

$$0\leqslant 2\left[f^T(Wx(t-d(t)))-x^T(t-d(t))W^TB\right]T\times[LWx(t-d(t))-f(Wx(t-d(t)))], \tag{23}$$

From (10)–(23), one can obtain that

$$\begin{aligned}
\dot{V}(x(t))\leqslant{}& 2\zeta^T(t)P\dot{\zeta}(t)+2f^T(Wx(t))\Lambda W\dot{x}(t)-(1-\dot{d}(t))\xi^T(t-d(t))Q\xi(t-d(t))+\xi^T(t)Z\xi(t)\\
&-\xi^T(t-h_1)(Z-R-Q)\xi(t-h_1))+h_1z^T(t)Xz(t)-\xi^T(t-h_2)R\xi(t-h_2))+h_{12}z^T(t-h_1)Sz(t-h_1)\\
&+\frac{h_1^2}{2}\dot{x}^T(t)U_1\dot{x}(t)+\frac{h_{12}^2}{2}\dot{x}^T(t-h_1)U_2\dot{x}(t-h_1)-\int_{t-h_1}^{t}z^T(s)Xz(s)\mathrm{d}s-\int_{t-d(t)}^{t-h_1}z^T(s)Sz(s)\mathrm{d}s-\int_{t-h_2}^{t-d(t)}z^T(s)Sz(s)\mathrm{d}s\\
&-\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta-\int_{-d(t)}^{-h_1}\int_{t+\theta}^{t-h_1}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta-\int_{-h_2}^{-d(t)}\int_{t+\theta}^{t-d(t)}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\\
&-(h_2-d(t))\int_{t-d(t)}^{t-h_1}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s+2\theta^T(t)M\left[x(t-h_1)-x(t-d(t))-\int_{t-d(t)}^{t-h_1}\dot{x}(s)\mathrm{d}s\right]\\
&+2\theta^T(t)N\left[x(t-d(t))-x(t-h_2)-\int_{t-h_2}^{t-d(t)}\dot{x}(s)\mathrm{d}s\right]\\
&+2\theta^T(t)H\left[(d(t)-h_1)x(t-h_1)-\int_{t-d(t)}^{t-h_1}x(s)\mathrm{d}s-\int_{-d(t)}^{-h_1}\int_{t+\theta}^{t-h_1}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right]\\
&+2\theta^T(t)F\left[(h_2-d(t))x(t-d(t))-\int_{t-h_2}^{t-d(t)}x(s)\mathrm{d}s-\int_{-h_2}^{-d(t)}\int_{t+\theta}^{t-d(t)}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right]\\
&+2\left[f^T(Wx(t))-x^T(t)W^TB\right]K[LWx(t)-f(Wx(t))]\\
&+2\left[f^T(Wx(t-d(t)))-x^T(t-d(t))W^TB\right]T[LWx(t-d(t))-f(Wx(t-d(t)))].
\end{aligned} \tag{24}$$

Using Lemma 1, one can obtain

$$-\int_{t-h_1}^{t}z^T(s)Xz(s)\mathrm{d}s\leqslant-\frac{1}{h_1}\int_{t-h_1}^{t}z^T(s)\mathrm{d}sX\int_{t-h_1}^{t}z(s)\mathrm{d}s, \tag{25}$$

$$\begin{aligned}
-\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta&\leqslant-\frac{2}{h_1^2}\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\mathrm{d}s\mathrm{d}\theta U_1\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\\
&=-\frac{2}{h_1^2}\left(h_1x^T(t)-\int_{t-h_1}^{t}x^T(s)\mathrm{d}s\right)U_1\left(h_1x(t)-\int_{t-h_1}^{t}x(s)\mathrm{d}s\right),
\end{aligned} \tag{26}$$

$$-(h_2-d(t))\int_{t-d(t)}^{t-h_1}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\leqslant-\frac{h_2-d(t)}{h_{12}}\int_{t-d(t)}^{t-h_1}\dot{x}^T(s)\mathrm{d}sU_2\int_{t-d(t)}^{t-h_1}\dot{x}(s)\mathrm{d}s, \tag{27}$$

Denoting $E_1=[0\quad 0\quad 0\quad 0\quad I]$, it is easy to see that

$$2\int_{t-h_2}^{t-h_1}x^T(s)\mathrm{d}sE_1P\dot{\zeta}(t)=2\left[\int_{t-d(t)}^{t-h_1}x^T(s)\mathrm{d}s+\int_{t-h_2}^{t-d(t)}x^T(s)\mathrm{d}s\right]E_1P\dot{\zeta}(t). \tag{28}$$

Clearly,

$$2\int_{t-d(t)}^{t-h_1} x^T(s)\mathrm{d}sE_1P\dot{\zeta}(t) - 2\theta^T(t)H\int_{t-d(t)}^{t-h_1} x(s)\mathrm{d}s - 2\theta^T(t)M\int_{t-d(t)}^{t-h_1} \dot{x}(s)\mathrm{d}s \leqslant (d(t)-h_1)\hat{\theta}^T(t)\begin{bmatrix}\Upsilon_1 & -\hat{M}\end{bmatrix}S^{-1}\begin{bmatrix}\Upsilon_1 & -\hat{M}\end{bmatrix}^T\hat{\theta}(t) + \int_{t-d(t)}^{t-h_1} z^T(s)Sz(s)\mathrm{d}s, \tag{29}$$

$$2\int_{t-h_2}^{t-d(t)} x^T(s)\mathrm{d}sE_1P\dot{\zeta}(t) - 2\theta^T(t)F\int_{t-h_2}^{t-d(t)} x(s)\mathrm{d}s - 2\theta^T(t)N\int_{t-h_2}^{t-d(t)} \dot{x}(s)\mathrm{d}s \leqslant (h_2-d(t))\hat{\theta}^T(t)\begin{bmatrix}\Upsilon_2 & -\hat{N}\end{bmatrix}S^{-1}\begin{bmatrix}\Upsilon_2 & -\hat{N}\end{bmatrix}^T\hat{\theta}(t) + \int_{t-h_2}^{t-d(t)} z^T(s)Sz(s)\mathrm{d}s, \tag{30}$$

$$-2\theta^T(t)F\int_{-h_2}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}(s)\mathrm{d}s\mathrm{d}\theta \leqslant \frac{(h_2-d(t))^2}{2}\hat{\theta}^T(t)\hat{F}U_2^{-1}\hat{F}^T\hat{\theta}(t) + \int_{-h_2}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \tag{31}$$

$$-2\theta^T(t)H\int_{-d(t)}^{-h_1}\int_{t+\theta}^{t-h_1} \dot{x}(s)\mathrm{d}s\mathrm{d}\theta \leqslant \frac{(d(t)-h_1)^2}{2}\hat{\theta}^T(t)\hat{H}U_2^{-1}\hat{H}^T\hat{\theta}(t) + \int_{-d(t)}^{-h_1}\int_{t+\theta}^{t-h_1} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \tag{32}$$

From (24)–(32), after some simple mathematical manipulations, it can be obtained that

$$\dot{V}(x(t)) \leqslant \hat{\theta}^T(t)\Xi_{d(t)}\hat{\theta}(t) \tag{33}$$

where $\Xi_{d(t)} = \Xi + (h_2-d(t))\hat{F}E_2 + (d(t)-h_1)\hat{H}E_3 - \frac{h_2-d(t)}{h_{12}}(E_3-E_2)U_2(E_3-E_2)^T + A_c^TYA_c + (d(t)-h_1)\begin{bmatrix}\Upsilon_1 & -\hat{M}\end{bmatrix}S^{-1}\begin{bmatrix}\Upsilon_1 & -\hat{M}\end{bmatrix}^T + (h_2-d(t))\begin{bmatrix}\Upsilon_2 & -\hat{N}\end{bmatrix}\times S^{-1}\begin{bmatrix}\Upsilon_2 & -\hat{N}\end{bmatrix}^T + \frac{(h_2-d(t))^2}{2}\hat{F}U_2^{-1}\hat{F}^T + \frac{(d(t)-h_1)^2}{2}\hat{H}U_2^{-1}\hat{H}^T$.

It is clear that $\hat{\theta}^T(t)\Xi_{d(t)}\hat{\theta}(t)$ is a convex quadratic function for $d(t)$ since its second order derivative with respect to $d(t)$ is $\hat{F}U_2^{-1}\hat{F}^T + \hat{H}U_2^{-1}\hat{H}^T \geqslant 0$. Therefore, according to Lemma 1 in [36], $\hat{\theta}^T(t)\Xi_{d(t)}\hat{\theta}(t) < 0$ is equivalent to

$$\Xi_{d(t)}|_{d(t)=h_1} < 0, \tag{34}$$

$$\Xi_{d(t)}|_{d(t)=h_2} < 0. \tag{35}$$

Using Schur complement, (34), (35) is equivalent to (36), (37), respectively. Therefore, if (36), (37) are satisfied, system (5) is asymptotically stable according to Lyapunov stability theory. □

Remark 3. By introducing a new Lyapunov functional (9), a less conservative stability condition is derived in Theorem 2. It should be noted that Lyapunov functional (9) contains a new double integral term $\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1} z^T(s)Sz(s)\mathrm{d}s\mathrm{d}\theta$ and a new triple integral term $\int_{-h_2}^{-h_1}\int_{\theta}^{-h_1}\int_{t+\lambda}^{t-h_1} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta$. While in our previous work [33], a Lyapunov functional containing a double integral term $\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} z^T(s)Sz(s)\mathrm{d}s\mathrm{d}\theta$ and a triple integral term $\int_{-h_2}^{-h_1}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta$ was introduced. After a careful comparison, we can see clearly that information about the lower bound of the time-varying delay d (t) are used in the inner integral upper limits of the double integral term and the triple integral term in Lyapunov functional (9). More specifically, the inner integral upper limits of $\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1} z^T(s)Sz(s)\mathrm{d}s\mathrm{d}\theta$ is '$t-h_1$' but not 't' and the integral upper limits of s and λ of $\int_{-h_2}^{-h_1}\int_{\theta}^{-h_1}\int_{t+\lambda}^{t-h_1} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta$ are '$t-h_1$' and '$-h_1$' respectively but not 't' and '0'. It is clear that information about the lower bound of the time-varying delay d (t) are more sufficiently used in our Lyapunov functional. Therefore, Lyapunov functional (9) may lead to less conservative results. In the next section, some numerical examples are given to confirm that results in this paper are less conservative than some existing ones.

Remark 4. There will be some integral terms and double integral terms in the derivative of the Lyapunov functional (9) since some double integral terms and triple integral terms are introduced in Lyapunov functional (9). For $-\int_{t-h_2}^{t-h_1} z^T(s)Xz(s)\mathrm{d}s$ and $-\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$, after being divided into some parts as shown in (15) and (17), some free-weighing matrices are introduced to estimate them. This is because that unlike the integral inequality method, free-weighing matrices method does not enlarge $-\frac{d(t)-h_1}{h_2-d(t)}$ as $-\frac{d(t)-h_1}{h_{12}}$ and consequently leads to less conservative results. While for $-\int_{t-h_1}^{t} z^T(s)Xz(s)\mathrm{d}s$ and $-\int_{-h_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$, the integral inequality method are used to cope with them. This is because the two terms only involve information about the lower bound of time-varying delay $d(t)$ and for such terms the integral inequality method gives the same level performance as free-weighting matrices method but with less decision variables.

In some circumstances, the information about the derivative of the delay may not be available. For this case, the following corollary can be obtained from Theorem 2 by setting $Q = 0$.

Table 1
Upper bounds of the delay for different h_1 and μ.

h_1	Methods	$\mu = 0.3$	$\mu = 0.5$	$\mu = 0.9$	μ unknown
0.1	Zuo et al. [28]	0.3900	0.2669	0.2668	0.2668
	Wu et al. [29]	0.3912	0.2678	0.2677	0.2677
	Our results	0.4249	0.3014	0.2857	0.2857
0.3	Zuo et al. [28]	0.4662	0.3996	0.3996	0.3996
	Wu et al. [29]	0.4662	0.4007	0.4007	0.4007
	Our results	0.5147	0.4134	0.4134	0.4134
0.5	Zuo et al. [28]	0.5640	0.5640	0.5640	0.5640
	Wu et al. [29]	0.5643	0.5643	0.5643	0.5643
	Our results	0.5743	0.5743	0.5743	0.5743

Table 2
Upper bounds of the delay for different μ and h_1.

h_1	Methods	$\mu = 0.1$	$\mu = 0.2$	$\mu = 0.3$	$\mu = 0.9$	μ unknown
0.1	Zuo et al. [28]	0.8402	0.6551	0.5493	0.3289	0.3289
	Wu et al. [29]	0.8402	0.6551	0.5493	0.3338	0.3338
	Our results	0.9282	0.7128	0.5891	0.3399	0.3399
0.5	Zuo et al. [28]	0.8402	0.6551	0.5880	0.5880	0.5880
	Wu et al. [29]	0.8402	0.6551	0.5886	0.5885	0.5885
	Our results	1.0497	0.7515	0.6021	0.6021	0.6021

Corollary 5. *For given scalars $0 < h_1 < h_2$ and μ, system (5) is asymptotically stable for any time-varying delay satisfying (2) if there exist matrices $P = [P_{ij}]_{5\times 5} > 0, Z = [Z_{ij}]_{3\times 3} \geqslant 0, R = [R_{ij}]_{3\times 3} \geqslant 0, X = [X_{ij}]_{2\times 2} \geqslant 0, S = [S_{ij}]_{2\times 2} \geqslant 0, U_1 \geqslant 0, U_2 \geqslant 0$, non-negative diagonal matrices K, T and Λ and any matrices $M = \begin{bmatrix} M_1^T & M_2^T & M_3^T \end{bmatrix}^T$, $N = \begin{bmatrix} N_1^T & N_2^T & N_3^T \end{bmatrix}^T, H = \begin{bmatrix} H_1^T & H_2^T & H_3^T \end{bmatrix}^T$, $F = \begin{bmatrix} F_1^T & F_2^T & F_3^T \end{bmatrix}^T$ with appropriate dimensions such that the following LMIs holds:*

$$\begin{bmatrix} \hat{\Delta} & h_{12}\Upsilon_1 & -h_{12}\hat{M} & \frac{h_{12}^2}{2}\hat{H} & A_c^T Y \\ * & -h_{12}S_{11} & -h_{12}S_{12} & 0 & 0 \\ * & * & -h_{12}S_{22} & 0 & 0 \\ * & * & * & -\frac{h_{12}^2}{2}U_2 & 0 \\ * & * & * & * & -Y \end{bmatrix} < 0, \tag{36}$$

$$\begin{bmatrix} \hat{\Theta} & h_{12}\Upsilon_2 & -h_{12}\hat{N} & \frac{h_{12}^2}{2}\hat{F} & A_c^T Y \\ * & -h_{12}S_{11} & -h_{12}S_{12} & 0 & 0 \\ * & * & -h_{12}S_{22} & 0 & 0 \\ * & * & * & -\frac{h_{12}^2}{2}U_2 & 0 \\ * & * & * & * & -Y \end{bmatrix} < 0, \tag{37}$$

$\hat{\Delta}$ and $\hat{\Theta}$ can be obtained respectively from Δ and Θ by setting $Q = 0$ and the other symbols are the same as those defined in Theorem 2.

4. Numerical examples

In this section, two numerical examples are given to illustrate the effectiveness of the proposed method, that is, the method in this paper can yield less conservative results than some existing ones.

Example 1. Consider the following delayed static recurrent neural network with [26]

$$A = \mathrm{diag}\{7.0214, 7.4367\}$$
$$W = \begin{bmatrix} -6.4993 & -12.0275 \\ -0.6867 & 5.6614 \end{bmatrix}$$
$$L = \mathrm{diag}\{l_1, l_2\} = \mathrm{diag}\{1, 1\}, \quad B = 0$$

The objective is to calculate the upper bound of the delay for given μ and h_1. Stability conditions proposed in this paper are implemented on an Intel (R) Core (TM) i5 processor at 2.40 GHz using Matlab LMI toolbox and the computation time is about 30.5s. Both the results obtained in [28,29] and the results obtained using the method proposed in this paper are listed in Table 1. It is clear that our results are significant better than those in [28,29], that is, much bigger upper bounds of the delay can be obtained in this paper.

Example 2. Consider the following delayed static recurrent neural network with [28]

$$A = \mathrm{diag}\{7.3458, 6.9987, 5.5949\}$$
$$W = \begin{bmatrix} 13.6014 & -2.9616 & -0.6936 \\ 7.4736 & 21.6810 & 3.2100 \\ 0.7290 & -2.6334 & -20.1300 \end{bmatrix},$$
$$L = \mathrm{diag}\{l_1, l_2, l_3\} = \mathrm{diag}\{0.3680, 0.1795, 0.2876\}, \quad B = 0,$$

Stability conditions proposed in this paper are implemented on an Intel (R) Core (TM) i5 processor at 2.40 GHz using Matlab LMI toolbox and the computation time is about 74.5s. The corresponding upper bounds of the delay for various given μ and h_1 calculated by Theorem 2 and Corollary 5 are listed in Table 2. For the purpose of comparison, results in [28,29] are also listed in Table 2. It is easy to see that our results are much less conservative than those in [28,29].

5. Conclusion

In this paper, the stability analysis problem of static recurrent neural networks with interval time-varying delay has been investigated. A new augmented Lyapunov functional which fully uses the information about the lower bound of the delay and contains some new double integral terms and triple-integral terms has been introduced. A new method to estimate the derivative of the Lyapunov functional has been proposed and some less conservative stability criteria have been obtained. Numerical examples have illustrated the effectiveness of the proposed method.

Acknowledgements

This work was supported in part by Natural Science Foundation of China under Grant No. 61104097, Research Fund for the Doctoral Program of Higher Education of China under Grant No. 20111101120027, National Science Foundation for Distinguished Young Scholars of China under Grant No. 60925011, and Projects of Major International (Regional) Joint Research Program NSFC under Grant 61120106010.

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