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Observer-based multiple-model adaptive output feedback control for a class of nonlinear systems

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Abstract

In this paper a systematic observer-based multiple-model adaptive controller design method is proposed for Lipschitz nonlinear systems. By introducing a compensator in the observer-based controller, the uncertainty due to the estimation error is decreased and the steady-state response is improved significantly. In order to deal with the uncertainty of system dynamics, a multiple-model switching scheme is introduced to improve the transient performance. A state-dependent dwell-time-based switching logic is used to ensure the asymptotic stability as it can cancel the possible increase of Lyapunov function in each switching. A simulation result is given to demonstrate the effectiveness of the proposed method.

Keywords

Adaptive control, nonlinear systems, nonlinear observer, multiple model, switching

Introduction

Adaptive control has been studied for some decades and a lot of literature can be found in this area. It can achieve good performance in time-invariant systems, linear or nonlinear (Astrom and Wittenmark, 1995; Krsti'c et al., 1995). It is well accepted that when the uncertainty is small, classical adaptive control can provide satisfactory closed-loop performance. However, classical adaptive control has some basic limitations, which may fail to ensure performance (Anderson and Dehghani, 2008), such as a large uncertainty due to varying operating conditions or unexpected changes in system dynamics.

Numerous efforts have been made in recent years to deal with control problems with large uncertainty. One example, involving multiple models, was reported in the 1990s. Generally, the multiple-model adaptive control (MMAC) scheme includes N parallel candidate models, a supervisor and a set of controllers. The concept of MMAC (Anderson et al., 2001a; Hespanha et al., 2001) is that there is an unknown plant P which belongs to a set $\mathscr{P}$, usually not a finite set. A set of controllers $\{C_1, C_2, \ldots\}$ is available and assumed to have the property that each plant in the set $\mathscr{P}$ will be satisfactorily controlled by at least one of the controllers C_i. The supervisor decides which controller can be applied on the plant. For the working principle of MMAC (Anderson et al., 2001a; Hespanha et al., 2001), the plant has an output y according to the input u, while the candidate plant I_i has an output $\hat{y}_i$. The disparity between y and $\hat{y}_i$ can help to make the decision which candidate plant is nearest to the real plant and then the corresponding controller is applied on the plant.

The idea of using multiple models in control has existed for a long time. The multiple Kalman filter was first introduced, in Athans et al. (1977), to improve the accuracy of the state estimate in control problems. Later, switching was combined with multiple models. The first example of using switching and tuning in multiple models for adaptive control was introduced in Narendra and Balakrishnan (1992) and was further studied by the same authors in 1994 and 1997. In Narendra and Balakrishnan (1994), the basic stability analysis for using multiple models for linear time-invariant systems was given for both fixed and adaptive models, and laid the foundations of this area. In Anderson et al. (2001a), the analysis of choosing the candidate model set and the corresponding controller set was given, and the stability analysis based on the *scale-independent* switching logic was proposed in Hespanha et al. (2001). In addition, the noise, disturbance and unmodelled dynamics were considered in both cases whether $\mathscr{P}$ was finite or infinite. In order to avoid instability due to the sudden change from one controller to another, some constraints on the switching logic were put forward in Anderson et al. (2001b). In Fekri et al. (2004a, 2004b) and Kuipers and Ioannou (2010), the mixed-μ synthesis design was used to improve the robustness of the system. Another breakpoint here was smooth switching logic, where a combination of controllers is used instead of a single controller, to avoid the chattering that results from the sudden change of controller. In Narendra and Han (2011) and Han and Narendra (2012) a new concept regarding MMAC was

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introduced, called 'second level adaptation'. This new approach provides an estimate which depends on the collective output of all the models, and the plant can be viewed as a time-varying convex combination of the estimates. It was proved that controllers designed based on this scheme can result in stable closed-loop systems.

Until now, the literature about MMAC has mainly focussed on linear systems, with few contributions about nonlinear systems. In Narendra and George (2002), an analysis of using multiple models to improve the transient performance of nonlinear systems based on switching and tuning was presented. In Ye (2008), a parametric-strict-feedback system was considered, and a design method for candidate models was proposed. It was proved that the switching stopped in finite time, which could sustain the stability.

Unfortunately, in the above literature all the system's states are assumed to be available, which is not always true in real systems. In this paper, an observer-based multiple-model adaptive control method is proposed for Lipschitz nonlinear systems. An adaptive observer is designed to estimate the unmeasurable states of the system. A compensator is introduced in the observer-based controller to decrease the effect of the uncertainty due to the estimation error. In order to deal with the uncertainty of system dynamics, a multiple-model switching scheme is introduced to improve the transient performance. The asymptotical stability is guaranteed by the state-dependent dwell-time switching logic.

In the remainder of this paper more details about the proposed MMAC design methodology are presented. After formulating the control problem in Section 2, the single model controller design based on the observer and compensator is presented in Section 3. In Section 4 the state-dependent dwell-time-based multiple-model switching logic is presented. A simulation example is given in Section 5, and finally the paper is concluded in Section 6.

Problem formulation

Consider the following nonlinear system

$$\Sigma : \begin{cases} \dot{x}(t) = f(x(t)) + bF(x(t))\theta + g(x(t))u(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x \in R^{n\times 1}, u \in R$ and $y \in R$ are the system state, input and output, respectively. $f(x) \in R^{n\times 1}, F(x) \in R^{1\times m}$ and $g(x) \in R^{n\times 1}$ are known functions. $b \in R^{n\times 1}, C \in R^{1\times n}$ are known parameters and $\theta \in R^{m\times 1}$ is the unknown parameter.

Some assumptions about the system are made as follows:

- **A.1** when the states of the system are available and the parameter θ is known, there exists a bounded controller $\| u = \alpha(x(t), \theta) \| \leqslant \chi$, a positive function $\bar{V}_1$ and $k_1(\| x(t) \|) \in \mathscr{K}$ such that

$$\dot{\bar{V}}_1 = \frac{\partial \bar{V}_1}{\partial x}(f(x(t)) + bF(x(t))\theta + g(x(t))\alpha(x(t), \theta)) < -\chi_1 k_1(\| x(t) \|) \tag{2}$$

  i.e. $\bar{V}_1$ is a Lyapunov function for system Σ, which regulates the output of system Σ to 0.
- **A.2** $f(x(t)) \in C^1$, $F(x(t)), g(x(t))$ are locally Lipschitz and satisfy

$$\| f(x_1(t)) - f(x_2(t)) \| \leqslant \gamma_0 \| x_1(t) - x_2(t) \|$$
$$\| F(x_1(t)) - F(x_2(t)) \| \leqslant \gamma_1 \| x_1(t) - x_2(t) \|$$
$$\| g(x_1)(t) - g(x_2)(t) \| \leqslant \gamma_2 \| x_1(t) - x_2(t) \|$$

- **A.3** there exists a positive definite matrix $P \in R^{n\times n}$ such that (Cho and Rajamani, 1997)

$$b^T P = C_1 \tag{3}$$

  where each row of C_1 lies in span of rows of C, i.e. there exist a matrix H such that $C_1 = HC$.
- **A.4** the vector of unknown parameter θ is bounded in the sense

$$\| \theta \| \leqslant \gamma_3 \tag{4}$$

Remark 1. For A.1, the methodology of state feedback controller design when the system states are available has been thoroughly explored in recent years, and we do not pay much attention to this problem here. More details can be found in Krsti'c et al. (1995) and Khalil (1995).

The main purpose of this paper is to design an observer-based multiple-model adaptive output feedback controller to improve the performance of system Σ.

Single model adaptive controller design

Research on nonlinear adaptive observer design has resulted in great improvements in recent decades (Besançon, 2000). The commonly used observer for the system in equation (1) is of the following form

$$\dot{\hat{x}}(t) = f(\hat{x}(t)) + bF(\hat{x}(t))\hat{\theta}(t) + g(\hat{x}(t))u(t) + L(C\hat{x}(t) - y(t)) \tag{5}$$

Define $\tilde{x}(t) = \hat{x}(t) - x(t)$ and $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$, and the error system is

$$\begin{aligned} \dot{\tilde{x}}(t) = & f(\hat{x}(t)) - f(x(t)) + bF(\hat{x}(t))\hat{\theta}(t) - bF(x(t))\theta \\ & + g(\hat{x}(t))u(t) - g(x(t))u(t) + L(C\hat{x}(t) - y(t)) \end{aligned} \tag{6}$$

If the system is linear, the observer-based controller design is very simple because of the *separation principle* (Brezinski, 2002), which ensures that the observer design and controller design can be considered separately. However, the *separation principle* does not hold in nonlinear systems (Kokotovic and Arcak, 2001). So, the observer and controller should be designed together. By combining equations (1), (5) and (6), the following augmented system can be obtained

$$\Sigma_a : \begin{cases} \dot{Z}(t) = \begin{bmatrix} f(x(t)) + bF(x(t))\theta + g(x(t))u(t) \\ f(\hat{x}(t)) + bF(\hat{x}(t))\hat{\theta}(t) + g(\hat{x}(t))u(t) + L(C\hat{x}(t) - y(t)) \end{bmatrix} \\ Y(t) = \begin{bmatrix} Cx(t) \\ C\hat{x}(t) \end{bmatrix} \\ u(t) = \alpha(\hat{x}(t), \hat{\theta}(t)) + \beta(\hat{x}(t), \hat{\theta}(t)) \end{cases} \quad (7)$$

where $Z(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}$ and $Y(t) = \begin{bmatrix} y(t) \\ \hat{y}(t) \end{bmatrix}$.

As for the stability of the system in equation (7), we have the following theorem.

Theorem 1. Consider the augmented nonlinear system in equation (7) and the assumptions **A.1–A.4**. If the following conditions hold:

- there exists a positive definite matrix P and a matrix L such that

$$A^T P + PA + (\gamma + \gamma_1\gamma_3 \parallel b \parallel + \gamma_2\chi)I + (\gamma + \gamma_1\gamma_3 \parallel b \parallel + \gamma_2\chi)PP + C^T L^T P + PLC < 0 \quad (8)$$

with $A = f'(0)$ and γ is a positive parameter to be designed

- the parameter update law is

$$\dot{\hat{\theta}}(t) = -F(\hat{x}(t))^T b^T P\tilde{x}(t) = -F(\hat{x}(t))^T C_1\tilde{x}(t) = -F(\hat{x}(t))^T H\tilde{y}(t) \quad (9)$$

- the compensator is

$$\beta(\hat{x}(t), \hat{\theta}(t)) = -\left(\left(\frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\right)\right)^T \left(\frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\right)^{-1} \times \left(\frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\right)^T \times \frac{\partial V_1}{\partial \hat{x}(t)} \times L(C\hat{x}(t) - y(t)) \quad (10)$$

then all the signals are bounded. The states of the system, equation (5), and the estimation error tend to the attractive zone $\{(x(t), \hat{x}(t), \tilde{x}(t)) | \parallel x(t) \parallel < \kappa, \parallel \hat{x}(t) \parallel < \kappa, \tilde{x}(t) = 0\}$ asymptotically, where κ is a designed parameter.

Proof: Consider the Lyapunov function candidate $V_o = \tilde{x}^T(t)P\tilde{x}(t) + \tilde{\theta}^T(t)\tilde{\theta}(t)$ for the observer and $V = V_1 + V_o$ for the system in equation (7). Under assumption **A.1**, we have

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_o \\ &= \frac{\partial V_1}{\partial \hat{x}}(f(\hat{x}(t)) + bF(\hat{x}(t))\hat{\theta}(t) + g(\hat{x}(t))u + L(C\hat{x}(t) - y(t))) + \dot{V}_o \\ &< -\chi_1 k_1(\parallel \hat{x}(t) \parallel) + \frac{\partial V_1}{\partial \hat{x}} g(\hat{x}(t))\beta(\hat{x}(t), \hat{\theta}(t)) + \frac{\partial V_1}{\partial \hat{x}} L(C\hat{x}(t) - y(t)) + \dot{V}_o \end{aligned} \quad (11)$$

where V_1 has the same form as $\bar{V}_1$, but $\hat{x}$ is used instead of x.

First, the gain matrix L and parameter update law can be designed based on $\dot{V}_o$ as in Cho and Rajamani (1997)

$$\begin{aligned} \dot{V}_o &= \dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t) + 2\tilde{\theta}(t)\dot{\tilde{\theta}}(t) \\ &= (f(\hat{x}(t)) - f(x(t)) + bF(\hat{x}(t))\hat{\theta}(t) - bF(x(t))\theta \\ &+ g(\hat{x}(t))u(t) - g(x(t))u(t) \\ &+ L(C\hat{x}(t) - y(t)))^T P\tilde{x}(t) + \tilde{x}^T(t)P(f(\hat{x}(t)) - f(x(t)) \\ &+ bF(\hat{x}(t))\hat{\theta}(t) - bF(x(t))\theta \\ &+ g(\hat{x}(t))u(t) - g(x(t))u(t) + L(C\hat{x}(t) - y(t))) + 2\tilde{\theta}^T(t)\dot{\tilde{\theta}}(t) \\ &= (f(\hat{x}(t)) - f(x(t)))^T P\tilde{x}(t) + \tilde{x}^T(t)P(f(\hat{x}(t)) \\ &- f(x(t))) + (bF(\hat{x}(t))\theta - bF(x(t))\theta)^T P\tilde{x}(t) \\ &+ \tilde{x}^T(t)P(bF(\hat{x}(t))\theta - bF(x(t))\theta) + (g(\hat{x}(t))u(t) \\ &- g(x(t))u(t))^T P\tilde{x}(t) \\ &+ \tilde{x}^T(t)P(g(\hat{x}(t))u(t) - g(x(t))u(t)) \\ &+ (L(C\hat{x}(t) - y(t)))^T P\tilde{x}(t) + \tilde{x}^T(t)P(L(C\hat{x}(t) - y(t))) \\ &+ (bF(\hat{x}(t))\tilde{\theta}(t))^T P\tilde{x}(t) + \tilde{x}^T(t)P(bF(\hat{x}(t))\tilde{\theta}(t)) + 2\tilde{\theta}^T(t)\dot{\tilde{\theta}}(t) \end{aligned} \quad (12)$$

As $f \in C^1$ is locally Lipschitz, then it is possible to get the Taylor series of $f(x) = f(0) + f'(0)x + o(\parallel x \parallel)$. And therefore

$$\begin{aligned} &(f(\hat{x}(t)) - f(x(t)))^T P\tilde{x}(t) + \tilde{x}^T(t)P(f(\hat{x}(t)) - f(x(t))) \\ &= (A\tilde{x}(t) + o(\parallel \hat{x}(t) \parallel) - o(\parallel x(t) \parallel))^T P\tilde{x}(t) + \tilde{x}^T(t)P(A\tilde{x}(t) \\ &+ o(\parallel \hat{x}(t) \parallel) - o(\parallel x(t) \parallel)) \\ &\leqslant \tilde{x}^T(t)(A^T P + PA)\tilde{x}(t) + \gamma \parallel \tilde{x}(t) \parallel \cdot \parallel P\tilde{x}(t) \parallel \\ &+ \gamma \parallel \tilde{x}^T(t)P \parallel \cdot \parallel \tilde{x}(t) \parallel \\ &= \tilde{x}^T(t)(A^T P + PA)\tilde{x}(t) + \gamma \cdot 2 \parallel \tilde{x}(t) \parallel \cdot \parallel P\tilde{x}(t) \parallel \\ &\leqslant \tilde{x}^T(t)(A^T P + PA)\tilde{x}(t) + \gamma(\tilde{x}^T(t)\tilde{x}(t) + \tilde{x}^T(t)PP\tilde{x}(t)) \\ &= \tilde{x}^T(t)(A^T P + PA + \gamma I + \gamma PP)\tilde{x}(t) \end{aligned} \quad (13)$$

$$\begin{aligned} &(bF(\hat{x}(t))\theta - bF(x(t))\theta)^T P\tilde{x}(t) + \tilde{x}^T(t)P(bF(\hat{x}(t))\theta - bF(x(t))\theta) \\ &\leqslant \parallel (bF(\hat{x}(t))\theta - bF(x(t))\theta)^T \parallel \cdot \parallel P\tilde{x}(t) \parallel \\ &+ \parallel P\tilde{x}(t) \parallel \cdot \parallel (bF(\hat{x}(t))\theta - bF(x(t))\theta) \parallel \\ &\leqslant 2 \cdot \parallel b \parallel \cdot \parallel F(\hat{x}(t)) - F(x(t)) \parallel \cdot \parallel \theta \parallel \cdot \parallel P\tilde{x}(t) \parallel \\ &\leqslant 2 \cdot \parallel b \parallel \cdot \parallel \theta \parallel \cdot \gamma_1 \parallel \tilde{x}(t) \parallel \cdot \parallel P\tilde{x}(t) \parallel \\ &\leqslant \gamma_1 \parallel b \parallel \parallel \theta \parallel (\tilde{x}^T(t)\tilde{x}(t) + \tilde{x}^T(t)PP\tilde{x}(t)) \\ &\leqslant \gamma_1\gamma_3 \parallel b \parallel \tilde{x}^T(t)(I + PP)\tilde{x}(t) \end{aligned} \quad (14)$$

In the same way

$$(g(\hat{x}(t))u(t) - g(x(t))u(t))^T P\tilde{x}(t) + \tilde{x}^T(t)P(g(\hat{x}(t))u(t) - g(x(t))u(t)) \leqslant \gamma_2\chi\tilde{x}^T(t)(I + PP)\tilde{x}(t) \quad (15)$$

Therefore

$$\begin{aligned} \dot{V}_o &\leqslant \tilde{x}^T(t)(A^T P + PA + (\gamma + \gamma_1\gamma_3 \parallel b \parallel + \gamma_2\chi)I \\ &+ (\gamma + \gamma_1\gamma_3 \parallel b \parallel + \gamma_2\alpha)PP + C^T L^T P + PLC)\tilde{x}(t) \\ &+ (bF(\hat{x}(t))\tilde{\theta}(t))^T P\tilde{x}(t) + \tilde{x}^T(t)P(bF(\hat{x}(t))\tilde{\theta}(t)) + 2\tilde{\theta}^T(t)\dot{\tilde{\theta}}(t) \end{aligned} \quad (16)$$

If the parameter update law is chosen as equation (9) and the matrix inequality in equation (8) is satisfied, then there exists a sufficient small positive definite matrix Q such that

$$\dot{V}_o < -\tilde{x}^T(t)Q\tilde{x}(t) \tag{17}$$

Then

$$\begin{aligned}\dot{V} < & -\chi_1 k_1(\| \hat{x}(t) \|) + \frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\beta(\hat{x}(t), \hat{\theta}(t)) \\ & + \frac{\partial V_1}{\partial \hat{x}(t)} L(C\hat{x}(t) - y(t)) - \tilde{x}^T(t)Q\tilde{x}(t)\end{aligned} \tag{18}$$

If the compensator $\beta(\hat{x}(t), \hat{\theta}(t)) = 0$, the effect of $L(C\hat{x}(t) - y(t))$ cannot be eliminated, which sometimes degrades the system's performance. What we should do is reduce the effect of $L(C\hat{x}(t) - y(t))$ as much as possible. That is, the compensator $\beta(\hat{x}(t), \hat{\theta}(t))$ is designed to minimize $\| \frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\beta(\hat{x}(t), \hat{\theta}(t)) + \frac{\partial V_1}{\partial \hat{x}(t)} L(C\hat{x}(t) - y(t)) \|$. Denote

$$r = \| \frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\beta(\hat{x}(t), \hat{\theta}(t)) + \frac{\partial V_1}{\partial \hat{x}(t)} L(C\hat{x}(t) - y(t)) \| \tag{19}$$

and the optimum β is given by

$$\begin{aligned}\beta(\hat{x}(t), \hat{\theta}(t)) = & -\left(\left(\frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\right)\right)^T \left(\frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\right)\right)^{-1} \\ & \times \left(\frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\right)^T \times \frac{\partial V_1}{\partial \hat{x}(t)} \times L(C\hat{x}(t) - y(t))\end{aligned} \tag{20}$$

Under this condition, $\| \frac{\partial V_1}{\partial \hat{x}(t)} g(\hat{x}(t))\beta(\hat{x}(t), \hat{\theta}(t)) + \frac{\partial V_1}{\partial \hat{x}(t)} L(C\hat{x}(t) - y(t)) \|$ reaches its minimum value denoted as χ_2. Therefore

$$\dot{V} < -\chi_1 k_1(\| \hat{x}(t) \|) - \tilde{x}^T(t)Q\tilde{x}(t) + \chi_2 \tag{21}$$

Solving the following equation and using the property $k_1(\| \hat{x} \|) \in \mathscr{K}$, it follows that when $\| \hat{x} \| > \kappa, \dot{V} < -\tilde{x}^T Q\tilde{x}$, where κ is a scalar and satisfies

$$-\chi_1 k_1(\kappa) + \chi_2 = 0 \tag{22}$$

Using the Lyapunov stability theory (Khalil, 1995) it is clear that $x, \hat{x}, \tilde{x}, \theta, \tilde{\theta}$ are bounded and the attractive zone $\{(x, \hat{x}, \tilde{x}) | \| x \| < \kappa, \| \hat{x} \| < \kappa, \tilde{x} = 0\}$ is asymptotically stable.

Remark 2. According to the equivalence of matrix norms, the norm used in this paper is $\| A \| = max(|a_{ij}|)$ if A is a matrix and $\| B \| = \sqrt{B^T B}$ if B is a vector.

Remark 3. For the parameter γ, it can be easily obtained by $\| o(\| \hat{x} \|) - o(\| x \|) \| - \| A\tilde{x} \| \leqslant \| f(\hat{x}) - f(x) \| = \| A\tilde{x} + o(\| \hat{x} \|) - o(\| x \|) \| \leqslant \gamma_0 \| \tilde{x} \|$ and $\| o(\| \hat{x} \|) - o(\| x \|) \| \leqslant \gamma_0 \| \tilde{x} \| + \| A \| \| \tilde{x} \|$.

Remark 4. For the matrix inequality in equation (8), the interior point method can be applied to obtain a feasible solution. For more details about the method of dealing with the inequality constraint $b^T P = C_1$, please refer to Cho and Rajamani (1997).

Remark 5. Further, as in Cho and Rajamani (1997), $\int_0^\infty \dot{\tilde{x}} \mathrm{d}t = \lim_{t\to\infty} \tilde{x}(t) - \tilde{x}(0)$. In addition, it follows from equation (6) that $\dot{\tilde{x}}$ is bounded. Then it follows from Barbalat's lemma that $\dot{\tilde{x}} \to 0$ and $bF(x)\theta - bF(\hat{x})\hat{\theta} \to 0$. If $\exists \zeta_0, \zeta_1, \delta > 0$ such that the PE condition in equation (23) holds for all t_0, there is $\tilde{\theta} \to 0$

$$\zeta_0 I \leqslant \int_{t_0}^{t_0 + \delta} bF(x(\tau))F(x(\tau))^T b^T \mathrm{d}\tau \leqslant \zeta_1 I \tag{23}$$

Multiple model design and analysis

The general nonlinear multiple-model adaptive control methodology contains three parts: N parallel candidate models $\{I_{j=1}^N\}$, a set of nonlinear parameterized controllers $\{u(\hat{x}(t)), \hat{\theta}_j(t))\}$ and a dwell-time switching scheme determining which candidate model and corresponding controller should be applied on the plant. Here, two cases of a multiple-model scheme are considered:

(a) all the candidate models are adaptive
(b) $N-1$ candidate models are fixed and one candidate model is adaptive.

Now consider the design of the controller and candidate models in both cases:

Case a: When the models are all adaptive, the i^{th} model has the following form

$$I_i : \begin{cases} \dot{\hat{x}}_i(t) = f(\hat{x}_i(t)) + bF(\hat{x}_i(t))\hat{\theta}_i(t) + g(\hat{x}_i(t))u(t) + L\tilde{y}_i(t) \\ \hat{y}_i(t) = C\hat{x}_i(t) \\ u_i(t) = \alpha(\hat{x}_i(t), \hat{\theta}_i(t)) + \beta(\hat{x}_i(t), \hat{\theta}_i(t)) \end{cases} \tag{24}$$

where $\tilde{y}_i(t) = C\hat{x}_i(t) - y(t)$. Using the same analysis method in Section 3, the gain matrix L and compensator $\beta(\hat{x}_i(t), \hat{\theta}_i(t))$ will be easily obtained, which are the same as equation (7).

Case b: When $N-1$ models are fixed and one model is adaptive, the fixed models have the same form as equation (24). As the parameter is fixed, the update law is no longer necessary while the gain matrix L and compensator $\beta(\hat{x}(t), \hat{\theta}_i(t))$ are the same as the system in equation (7), which can be tested easily.

As for the switching scheme, it monitors a function for each candidate model and determines online which candidate model and controller should be applied on the plant. Denote the candidate error

$$\varepsilon_i(t) = y_i(t) - y(t) \tag{25}$$

where y_i is the output of the i^{th} candidate model and y is the output of the real system. A rational performance index is

$$J_{I_i} = \pi_1 \varepsilon_i(t)^2 + \pi_2 \int_0^t \varepsilon_i(\tau)^2 d\tau, \pi_1, \pi_2 > 0 \quad i = 1, 2, \cdots, N \tag{26}$$

which takes both the instantaneous and long-term candidate error into consideration. It was proved by Narendra and Balakrishnan (1994, 1997) and Narendra and George (2002) that this performance index leads to improved transient performance.

It is well known that stability is a key problem and must be considered in the switching scheme. The switching scheme and stability analysis is as follows:

Initialization: Choose a controller randomly and apply it on the plant, set I_c.

Switching scheme: Suppose the last switch occurs at t_1. Then for $t > t_1$, if $t - t_1 > \tau_D$, where τ_D is the dwell-time which will be determined later, and $min_{1 \leq i \leq N} J_{I_i} \leq J_{I_c}$, then set $I_{i^*} = arg\ min_{1 \leq i \leq N} J_{I_i}(t)$ and $I_c = I_{i^*}$.

Theorem 2. Suppose that the nonlinear multiple-model adaptive control scheme is proposed above. Then for any initial conditions, all signals in the system are bounded and the attractive zone $\{(x, \tilde{x}) |\, \| x \| < \kappa, \tilde{x} = 0\}$ is asymptotically stable.

Proof: The proof is presented separately for the two cases described earlier.

Case a (all adaptive models):

Suppose the j^{th} candidate model is active and the controller u_j is applied on the plant, then consider the performance of the i^{th} model.

Choose the following Lyapunov functions

$$V_i = V_{1i} + \tilde{x}_i^T(t) P \tilde{x}_i(t) + \tilde{\theta}_i^T(t) \tilde{\theta}_i(t) \tag{27}$$

where V_{1i} has the same form as equation (2) with the i^{th} model's corresponding controller $u_i(t)$, and

$$V_{i-j} = V_{1,i-j} + \tilde{x}_i^T(t) P \tilde{x}_i(t) + \tilde{\theta}_i^T(t) \tilde{\theta}_i(t) \tag{28}$$

with the j^{th} model's corresponding controller $u_j(t)$. Then

$$\begin{aligned}
\dot{V}_{i-j} &= \frac{\partial V_{1,i-j}}{\partial \hat{x}_i}(f(\hat{x}_i(t)) + bF(\hat{x}_i(t))\hat{\theta}_i(t) + g(\hat{x}_i(t))u_j(t) \\
&\quad + L(C\hat{x}_i(t) - y(t))) + 2\tilde{x}_i^T(t) P \dot{\tilde{x}}_i(t) + 2\tilde{\theta}_i^T(t)\dot{\tilde{\theta}}_i(t) \\
&= \frac{\partial V_{1,i-j}}{\partial \hat{x}_i}(f(\hat{x}_i(t)) + bF(\hat{x}_i(t))\hat{\theta}_i(t) + g(\hat{x}_i(t))u_i(t) \\
&\quad + L(C\hat{x}_i(t) - y(t))) \\
&\quad + 2\tilde{x}_i^T(t) P \dot{\tilde{x}}_i(t) + 2\tilde{\theta}_i^T(t)\dot{\tilde{\theta}}_i(t) + \frac{\partial V_{1,i-j}}{\partial \hat{x}_i} g(\hat{x}_i(t))u_j(t) \\
&\quad - \frac{\partial V_{1,i-j}}{\partial \hat{x}_i} g(\hat{x}_i(t))u_i(t) \\
&= \dot{V}_i + \frac{\partial V_{1,i-j}}{\partial \hat{x}_i} g(\hat{x}_i(t))u_j(t) - \frac{\partial V_{1,i-j}}{\partial \hat{x}_i} g(\hat{x}_i(t))u_i(t)
\end{aligned} \tag{29}$$

As all the signals of the single model are bounded, it follows that

$$\dot{V}_{i-j} < -\alpha_1 k_1(\| x_i \|) + \chi_2 - \tilde{x}^T(t) Q \tilde{x}(t) + \chi_{i-j} \tag{30}$$

where χ_{i-j} is a positive constant which satisfies

$$\left\| \frac{\partial V_{1,i-j}}{\partial \hat{x}_i} g(\hat{x}_i(t))u_j(t) - \frac{\partial V_{1,i-j}}{\partial \hat{x}_i} g(\hat{x}_i(t))u_i(t) \right\| < \chi_{i-j} \tag{31}$$

From this equation it follows that when the j^{th} model is active and the controller u_j is applied on the i^{th} model, the estimation error and control error are decreasing or the increase is at least bounded. When the controller switches back to i, the estimation error and control error will decrease to a certain small bound region. Then by carefully designing the dwell-time, the increase can be cancelled.

Set $S = \{t_1, t_2, \cdots\}$, where t_i is the time when the switching occurs and $T_D = \{\tau_{D1}, \tau_{D2}, \tau_{D3}, \cdots\}$, where τ_{Di} is the dwell-time for the i^{th} switch. Suppose that controller u_i is switched into the system at time t_k while t_{k^*} is the last time that u_i was switched into the system. Set the dwell-time as τ_{Dk}. In order to cancel the increase, it has to satisfy

$$\int_{t_{k^*}}^{t_k + \tau_{Dk}} \dot{V}_i \, dt + \chi^* \times (k - k^*) < 0 \tag{32}$$

here $\chi^* = \max\{\chi_{i-j}, j = 1, 2, \cdots, N\}$

Further

$$\chi^* \times (k - k^*) < \int_{t_{k^*}}^{t_k + \tau_{Dk}} k_x(\| \hat{x}(t) \|) \, dt \tag{33}$$

By solving equation (33) the dwell-time τ_{Dk} for the k^{th} switching is obtained.

Then, by applying the stability analysis in Branicky (1998) it follows that the multiple-model switching system is stable. Further with Theorem 1, for any initial conditions, all signals in the system are bounded and the attractive zone $\{(x, \tilde{x}) |\, \| x \| < \kappa, \tilde{x} = 0\}$ is asymptotically stable.

Case b ($N - 1$ fixed models and one adaptive model):

In this case θ_i is fixed for $i \in \{1, 2, \cdots, N-1\}$ and the N^{th} model is adaptive. When the i^{th} fixed model is active and controller u_i is applied on the plant, using the analysis in *case a* it follows that the estimation error and control error of the adaptive model are decreasing or the increase is at least bounded. Then using the dwell-time design in equation (33) the error of the adaptive system can decrease to 0, which leads to a bounded J_N while $\{J_i, 1 \leq i \leq N-1\}$ grow unbounded. Hence, there exists a finite time such that the adaptive system is active for all $t > T$. Using the results in Theorem 1 it follows that for any initial conditions, all signals in the system are bounded and the attractive zone $\{(x, \tilde{x}) |\, \| x \| < \kappa, \tilde{x} = 0\}$ is asymptotically stable.

A simulation example

Consider the following nonlinear system

$$\begin{cases} \dot{x}_1(t) = x_2(t) + 0.4 \times \theta cos x_1(t) \\ \dot{x}_2(t) = -6 \sin x_1(t) - 8 \sin x_2(t) - 0.2 * \theta \cos, x_1(t) + u(t) \\ y = x_1(t) \end{cases} \tag{34}$$

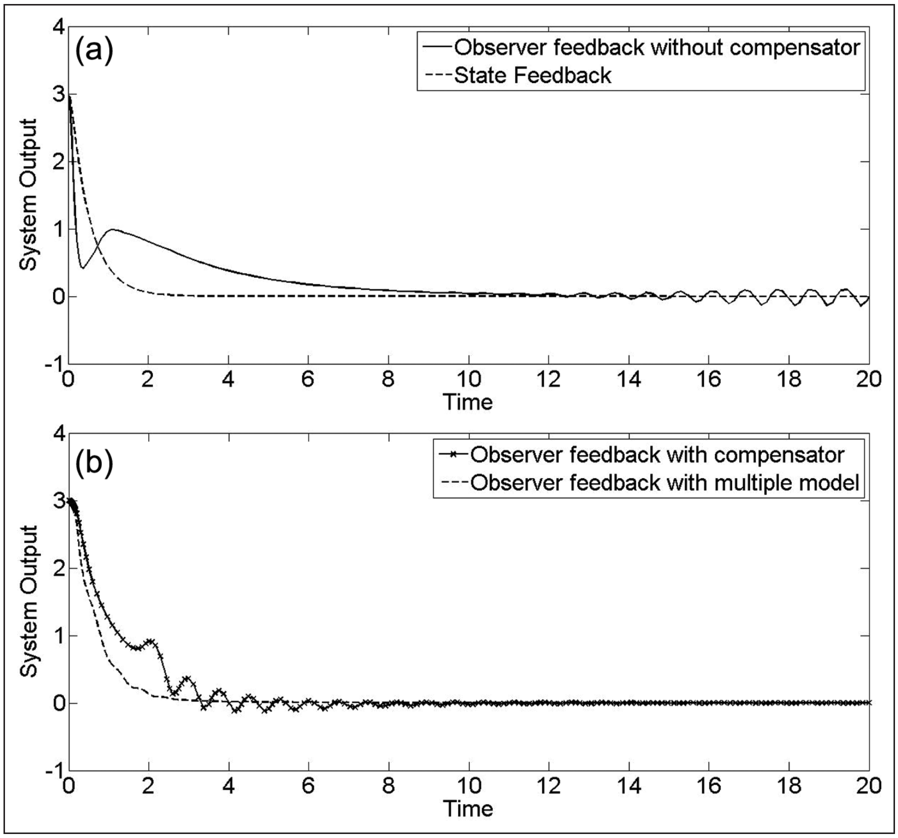

Figure 1. The system output.

$f(x(t)) = \begin{bmatrix} x_2(t) \\ -6\sin x_1(t) - 8\sin x_2(t) \end{bmatrix}, b = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}$, $F(x(t)) = \cos x_1(t), g(x(t)) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 0]$ and θ is the unknown parameter within the known uncertainty zone $\theta \in [0, 20]$. When all the states are available and θ is known, it is easy to design a state feedback control law u^* using the backstepping methods in Krsti'c et al. (1995) to regulate the output y to 0

$$u^*(t) = -x_1(t) - 20z_2(t) + 6\sin x_1(t) + 8\sin x_2(t) - 0.2\times\theta\cos x_1(t) + \dot{\alpha}_1(t) \tag{35}$$

where $\alpha_1(t) = -2 * x_1(t) - 0.4\times\theta\cos x_1(t)$ and $z_2(t) = x_1(t) - \alpha_1(t)$.

Furthermore, by solving the matrix inequality in equation (8) using the method in Cho and Rajamani (1997), we get

$$P = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}, L = \begin{bmatrix} -22.4850 \\ -4.1175 \end{bmatrix}$$

which satisfies $b^T P = C_1$. And the compensator is obtained from equation (20)

$$\beta(\hat{x}(t), \hat{\theta}(t)) = \frac{1}{|\hat{x}_2(t)|} L^T \hat{x}(t) \times \tilde{y}(t) \tag{36}$$

For the multiple-model design, nine identification models are available and the initial parameters are set as $\{0, 1, 3, 6, 9, 12, 15, 18, 20\}$. The parameters update as equation (9) and the dwell-time is obtained online with equation (33).

From Figure 1(a) we can see that when all the states are available and θ is known, the system's output under the control law u^* approaches zero fast and smoothly, which is shown with a dash line. When all the states are not available and θ is unknown, we can obtain a control law $\hat{u}^*$ replacing x and θ in u^* by $\hat{x}$ and $\hat{\theta}$, respectively. Using $\hat{u}^*$, the system's output is shown as a solid line in Figure 1(a) and we can see the system's output is still bounded, but there is a jitter in the steady state.

Applying the compensator in equation (36) on the basis of $\hat{u}^*$ without using the multiple-model scheme, the system's output is shown in Figure 1(b) by the solid line. It can be seen the steady-state performance of the system is improved. In order to deal with the uncertainty of system parameters and improve the transient performance, the multiple-model scheme and the compensator are used and the system's output is shown in Figure 1(b) by the dashed line. It can be seen that both the transient and steady-state performance is much improved. Figure 2 shows that the switching will stop in finite time, which is proved in Theorem 2.

Conclusion

A systematic algorithm for nonlinear adaptive control based on the observer and multiple models has been proposed in this paper. A compensator has been designed to improve the steady-state performance and a multiple-model switching scheme has

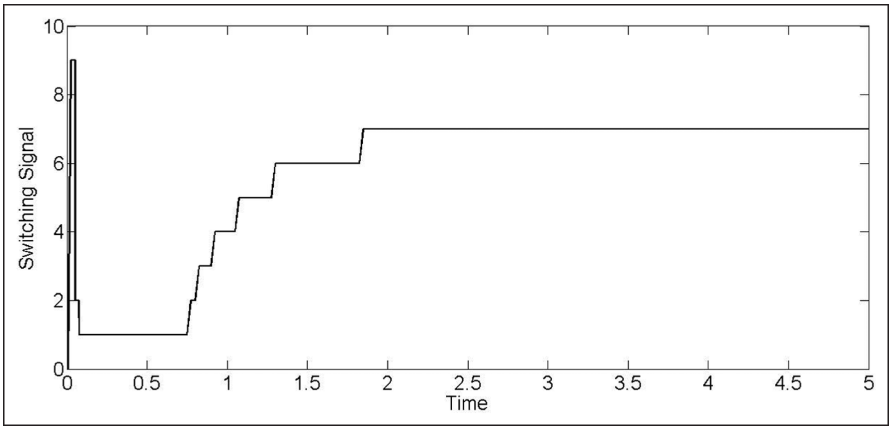

Figure 2. The switching signal of the multiple-model switching scheme.

been introduced to improve the transient performance. Furthermore, a state-dependent dwell-time scheme has been proposed to ensure the stability of the multiple-model switching system. It has been proved that the switching stops in finite time and stays on the adaptive system forever. A simulation example demonstrated the efficiency of the proposed method.

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