

Adaptive control of a class of nonlinear systems using multiple models with smooth controller

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SUMMARY

The idea of using multiple models to improve transient performance in adaptive control systems with large uncertainty or time varying parameters was introduced in 1990s. However, the commonly used scheme with switching has some potential drawbacks. In this paper, a new multiple model scheme is proposed for strict-feedback nonlinear systems. In order to avoid the possible chattering resulted from the controller's switching, a continuous controller based on the convex combination of parameter estimates of identification models is presented, which ensures the better use of the information of identification models than the switching scheme. Also, the number of necessary models is just one more than the dimension of the unknown system parameter, which is more practical. Simulation studies are presented to demonstrate the efficiency of the proposed scheme. Copyright © 2013 John Wiley & Sons, Ltd.

Received 3 December 2012; Revised 20 October 2013; Accepted 21 October 2013

KEY WORDS: multiple model; convex index; adaptive control; strict-feedback nonlinear system

1. INTRODUCTION

The classical adaptive control theory has been studied since 1960s, and a lot of literature can be found in this area [1–4]. As an efficient way to deal with time-invariant systems with uncertain parameters, it gains great improvements in recent decades. It is well accepted that when the uncertainty is small, the classical adaptive control can achieve satisfactory closed-loop performance. But when it comes to large uncertainties or time-varying systems, the classical adaptive control fails in some sense [5]. Then, multiple model adaptive control, an improved adaptive control scheme using multiple models, was proposed to deal with these limitations.

The general multiple model adaptive control scheme includes N parallel identification models, a controller set, and a supervisor logic. The identification models may be fixed or adaptive, with at least one model close enough to the real system. The controller set is designed to make sure that at least one of the controllers can achieve satisfactory performance for the system. The supervisor logic makes the decision of which identification model is best based on the performance of each model and then determines the system input.

The idea of using multiple models in control problems has existed for a long time. In [6], multiple Kalman filter was first introduced to improve the accuracy of the state estimate in control systems. The idea of using switching and tuning in multiple models for adaptive control was first introduced in [7] and got further studied in [8, 9], where both fixed and adaptive models were used in the identification model design and controller design. In [10], one possible method to design the

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model set was proposed, where the Vinnicombe metric and the controllable range of the robust controller were used to decide the permitted uncertainty of each identification model, but it was hard to extend this idea to high dimensional systems. It was demonstrated by extensive simulations [11, 12] that satisfactory performance could always be obtained if the number of identification model was large enough. In order to avoid the chattering resulted from the suddenly change of controller, some constraints were put on the switching logic in [13]. Scale-independent dwell-time switching, a well-developed switching logic for linear and nonlinear systems, was proposed in [14]. It was proved that there were finite switchings in finite time, and the average dwell time was adjustable in [15].

From a practical point of view, the multiple model scheme with switching has some potential shortcomings. First, the control signal is not continuous, even with a large dwell time. This discontinuity may lead to transient chattering of system performance. Second, the number of identification model is always large, especially for high-dimensional systems, as it is needed that at least one in the model set is sufficiently close to the real model. Third, there is no information communication between the model sets, and the supervisor logic only depends on the output differences between identification models. In [16–18], the combination of controllers based on the performance of each identification model was proposed, which ensured the smoothness of control signal. In [19], a new multiple model scheme was developed for linear systems considering the drawbacks mentioned earlier. For the model set, only $N + 1$ models were needed if the dimension of the unknown system parameter was N . It was proved that the unknown system parameter was always in the convex hull of the identification parameters if it was in the hull at the beginning, and the convex combination of identification errors was zero, which was used to design the second level adaptation for the convex indexes. Further, a virtual model, which was designed based on the identification parameters and convex indexes, was used to design the continuous controller.

In this paper, we mainly try to extend the idea in [19] to nonlinear systems. The main challenge is that when the system is nonlinear, the system parameter may not be in the convex hull of identification parameters even if it is in the hull at the beginning, which further leads to the problem that the convex combination of the identification errors is not zero. However, with certain identification model structure and parameter update rule as designed in this paper, the system parameter can stay in the convex hull all the time. As for the combination error, it is not always zero because of the nonlinearity and initial conditions, but the exponential convergence to zero is obtained in this paper. A continuous controller based on the combined information of all identification models is obtained to avoid the possible chattering. Using the convex hull property in [19], only $q + 1$ identification models are needed when the dimension of the unknown parameter is q . Then, the fixed identification model set is considered and the convex indexes are treated as the unknown parameter, which reduces the system uncertainty to a unit ball.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following strict-feedback nonlinear system:

$$\Sigma : \begin{cases} \dot{x}_i = x_{i+1} + \theta^T \varphi_i(\bar{x}_i), & i = 1, 2, \dots, n-1 \\ \dot{x}_n = u + \theta^T \varphi_n(x) \\ y = x_1 \end{cases} \quad (1)$$

where $\bar{x}_i = [x_1, x_2, \dots, x_i]$, $x = [x_1, x_2, \dots, x_n]^T \in R^n$ is the state vector; $u \in R$ and $y \in R$ are the system input and output, respectively; $\theta \in R^q$ is the unknown system parameter belonging to a known compact set \mathbf{S} and $\varphi_i(\bar{x}_i) \in R^q, i = 1, 2, \dots, n$ are known Lipschitz functions. The main purpose is to design a multiple model-based smooth controller to improve the transient performance and, meanwhile, ensure the stability.

For the multiple model scheme, $q + 1$ parallel identification models $\{I_s\}_{s=1}^{q+1}$ are needed. These models have the same structure, but with different initial parameter estimates $\theta_1(0), \theta_2(0), \dots, \theta_{q+1}(0)$, which satisfies that the compact set \mathbf{S} is within the convex hull of $\theta_i(0), i = 1, 2, \dots, q + 1$.

For design simplicity, the identification models are designed as follows:

$$I_s : \begin{cases} \dot{x}_{si} = -\lambda(x_{si} - x_i) + x_{s(i+1)} + \theta_s^T \varphi_i(\bar{x}_i), & i = 1, 2, \dots, n-1 \\ \dot{x}_{sn} = -\lambda(x_{sn} - x_n) + u + \theta_s^T \varphi_n(x) \\ y_s = x_{s1} \end{cases} \quad (2)$$

where $s = 1, 2, \dots, q+1$ and $\lambda > 0$ is a parameter to be designed.

Define the identification error as

$$e_{si} = x_{si} - x_i, \tilde{\theta}_s = \theta_s - \theta, i = 1, 2, \dots, n$$

we can obtain the identification error system as

$$\dot{e}_s = A e_s + \tilde{\theta}_s^T \varphi(x), A = \begin{pmatrix} -\lambda & 1 & 0 & \dots & 0 \\ 0 & -\lambda & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & -\lambda \end{pmatrix} \quad (3)$$

and the following Lemma can be easily obtained.

Lemma 1

If the initial parameter of the identification model is the same as that of the system model, the identification error tends to 0 exponentially if $\lambda > 1$.

The following facts and lemmas are needed in the derivation of main results.

Lemma 2 (Barbălat's Lemma [3])

If $\lim_{t \rightarrow \infty} f(t)$ exists and is finite, and $\dot{f}(t)$ is a uniformly continuous function, then $\lim_{t \rightarrow \infty} \dot{f}(t) = 0$.

Lemma 3 (PE condition [20])

Let $g : [0, \infty] \rightarrow R^n$ be a continuous differentiable function and $f : [0, \infty] \rightarrow R^n$ be a bounded piecewise continuous function. Further assume that there exist positive constants ϵ, t_0, T_0 such that, for any unit row vector \mathbf{c} of dimension n and any $t \geq t_0$

$$\frac{1}{T_0} \int_t^{t+T_0} |\mathbf{c}f(s)| ds \geq \epsilon.$$

Then, $\lim_{t \rightarrow \infty} g(t) = 0$ if $\lim_{t \rightarrow \infty} \dot{g}(t) = 0$ and $\lim_{t \rightarrow \infty} g^T(t)f(t) = 0$.

Fact 1

For $\forall \epsilon > 0$ and $\forall \rho \in R$, the following inequality holds

$$0 \leq |\rho| - \rho \cdot \tanh(\rho/\epsilon) \leq 0.2785\epsilon. \quad (4)$$

Fact 2

For $\forall \theta \in R^q$, there exist $q+1$ points, which can make a convex hull \mathfrak{F} and $\theta \in \mathfrak{F}$.

3. MULTIPLE MODEL-BASED CONTROLLER DESIGN

In this section, we consider the multiple model-based controller design for both adaptive and fixed identification model sets. First, the parameter update of adaptive identification models is presented. Then a smooth controller with the combination of identification parameters is designed based on backstepping and a simple filter. The adaptation of convex indexes is designed with the identification errors. Finally, when all the identification models are fixed, the convex indexes are treated as unknown parameters, and some existing methods are used to analyze the design.

3.1. Adaptive models

3.1.1. Controller design. Consider the error system (3), in order to make the identification model (2) approach the system (1), the Lyapunov function is chosen as follows:

$$V_{se} = \frac{1}{2}e_{s1}^2 + \dots + \frac{1}{2}e_{sn}^2 + \frac{1}{2}\tilde{\theta}_s^T \tilde{\theta}_s \tag{5}$$

and

$$\begin{aligned} \dot{V}_{se} &= -\lambda \sum_{i=1}^n e_{si}^2 + \sum_{i=1}^{n-1} e_{si} e_{s(i+1)} + \tilde{\theta}_s^T \left(\dot{\tilde{\theta}}_s + \sum_{i=1}^n e_{si} \varphi_i(\bar{x}_i) \right) \\ &\leq -(\lambda - 1) \sum_{i=1}^n e_{si}^2 - \frac{1}{2} \sum_{i=1}^{n-1} (e_{si} - e_{s(i+1)})^2 + \tilde{\theta}_s^T \left(\dot{\tilde{\theta}}_s + \sum_{i=1}^n e_{si} \varphi_i(\bar{x}_i) \right). \end{aligned}$$

Choose the parameter adaptation law as

$$\dot{\tilde{\theta}}_s = - \sum_{i=1}^n e_{si} \varphi_i(\bar{x}_i) \tag{6}$$

and it comes to the following lemma.

Lemma 4

If $\lambda > 1$ and the parameter adaptation law of identification model I_s is set as (6), the identification error tends to 0 exponentially.

Remark 1

According to (3) and Lemma 2, $e_s \rightarrow 0$ and \dot{e}_s is uniformly continuous, then $\dot{e}_s \rightarrow 0$, which implies $\tilde{\theta}_s^T \varphi(x) \rightarrow 0$. Also, $\tilde{\theta}_s \rightarrow 0$ in (6) as $\varphi_i(\bar{x}_i), i = 1, 2, \dots, n$ are Lipschitz and bounded. So, if for any t , there exist ϵ_0, t_0, T_0 and a unit vector c such that $\forall t > t_0, \frac{1}{T_0} \int_t^{t+T_0} |c\varphi(\tau)| d\tau > \epsilon_0$, it follows from Lemma 3 that $\tilde{\theta}_s \rightarrow 0$.

Then for model I_s , the controller is designed in the following steps:

Step 1,

$$\dot{x}_{s1} = -\lambda(x_{s1} - x_1) + x_{s2} + \theta_s^T \varphi_1(x_1).$$

Let $z_{s1} = x_{s1}, z_{s2} = x_{s2} - \alpha_{s1}$, and $\alpha_{s1} = \lambda(x_{s1} - x_1) - k_{s1}z_{s1} - \theta_s^T \varphi_1(x_1)$, then

$$\begin{aligned} V_{s1} &= \frac{1}{2}z_{s1}^2 \\ \dot{V}_{s1} &= -k_{s1}z_{s1}^2 + z_{s1}z_{s2} \end{aligned}$$

Step 2,

$$\dot{x}_{s2} = -\lambda(x_{s2} - x_2) + x_{s3} + \theta_s^T \varphi_2(x_1, x_2).$$

Let $z_{s3} = x_{s3} - \alpha_{s2}$, it comes that

$$\dot{z}_{s2} = -\lambda(x_{s2} - x_2) + z_{s3} + \alpha_{s2} + \theta_s^T \varphi_2(x_1, x_2) - \dot{\alpha}_{s1}$$

and

$$\begin{aligned} \dot{\alpha}_{s1} &= \frac{\partial \alpha_{s1}}{\partial x_1} \dot{x}_1 + \frac{\partial \alpha_{s1}}{\partial x_{s1}} \dot{x}_{s1} + \frac{\partial \alpha_{s1}}{\partial \theta_s} \dot{\theta}_s + \frac{\partial \alpha_{s1}}{\partial z_{s1}} \dot{z}_{s1} \\ &= \frac{\partial \alpha_{s1}}{\partial x_1} (x_2 + \theta \varphi_1(x_1, t)) + \frac{\partial \alpha_{s1}}{\partial x_{s1}} \dot{x}_{s1} + \frac{\partial \alpha_{s1}}{\partial \theta_s} \dot{\theta}_s + \frac{\partial \alpha_{s1}}{\partial z_{s1}} \dot{z}_{s1}. \end{aligned}$$

As $\dot{\alpha}_{s1}$ contains the unknown parameter θ , it is not possible to obtain the exact value of $\dot{\alpha}_{s1}$. Also, the problem of term explosion in back-stepping makes the design complex. In order to avoid these problems, the following filter is introduced:

$$\begin{cases} \tau_{s2}\dot{x}_{s2f} + x_{s2f} = \alpha_{s1} - \beta_{s2}\tau_{s2}\tanh\left(\frac{\beta_{s2}(y_{s2f} + z_{s2})}{\varsigma_{s2}}\right) \\ y_{s2f} = x_{s2f} - \alpha_{s1} \end{cases} \quad (7)$$

Let $\alpha_{s2} = \lambda(x_{s2} - x_2) - z_{s1} - k_{s2}z_{s2} - \theta_s^T \varphi_2(x_1, x_2) + \dot{x}_{s2f}$ and

$$V_{s2} = V_{s1} + \frac{1}{2}z_{s2}^2 + \frac{1}{2}y_{s2f}^2$$

then,

$$\begin{aligned} \dot{V}_{s2} &= \dot{V}_{s1} + z_{s2}(-z_{s1} - k_{s2}z_{s2} + z_{s3} + \dot{y}_{s2f}) + y_{s2f}\dot{y}_{s2f} \\ &= -k_{s1}z_{s1}^2 - k_{s2}z_{s2}^2 + z_{s2}z_{s3} + z_{s2}\dot{y}_{s2f} + y_{s2f}\dot{y}_{s2f} \end{aligned}$$

and

$$\begin{aligned} & z_{s2}\dot{y}_{s2f} + y_{s2f}\dot{y}_{s2f} \\ &= -z_{s2}\left(\frac{1}{\tau_{s2}}y_{s2f} + \beta_{s2}\tanh\left(\frac{\beta_{s2}(y_{s2f} + z_{s2})}{\varsigma_{s2}}\right) + \dot{\alpha}_{s1}\right) \\ & \quad - y_{s2f}\left(\frac{1}{\tau_{s2}}y_{s2f} + \beta_{s2}\tanh\left(\frac{\beta_{s2}(y_{s2f} + z_{s2})}{\varsigma_{s2}}\right) + \dot{\alpha}_{s1}\right) \\ &= -\frac{1}{\tau_{s2}}z_{s2}y_{s2f} - \frac{1}{\tau_{s2}}y_{s2f}^2 - (z_{s2} + y_{s2f})\dot{\alpha}_{s1} - \beta_{s2}(z_{s2} + y_{s2f})\tanh\left(\frac{\beta_{s2}(y_{s2f} + z_{s2})}{\varsigma_{s2}}\right) \\ &\leq -\frac{1}{\tau_{s2}}z_{s2}y_{s2f} - \frac{1}{\tau_{s2}}y_{s2f}^2 + |(z_{s2} + y_{s2f})| \cdot |\dot{\alpha}_{s1}| - \beta_{s2}(z_{s2} + y_{s2f})\tanh\left(\frac{\beta_{s2}(y_{s2f} + z_{s2})}{\varsigma_{s2}}\right) \end{aligned}$$

if $\beta_{s2} > \max|\dot{\alpha}_{s1}|$, it follows from Fact 1 that

$$z_{s2}\dot{y}_{s2f} + y_{s2f}\dot{y}_{s2f} \leq -\frac{1}{\tau_{s2}}z_{s2}y_{s2f} - \frac{1}{\tau_{s2}}y_{s2f}^2 + 0.2785\varsigma_{s2}$$

and

$$\begin{aligned} \dot{V}_{s2} &\leq -k_{s1}z_{s1}^2 - k_{s2}z_{s2}^2 + z_{s2}z_{s3} - \frac{1}{\tau_{s2}}z_{s2}y_{s2f} - \frac{1}{\tau_{s2}}y_{s2f}^2 + 0.2785\varsigma_{s2} \\ &\leq -k_{s1}z_{s1}^2 - \left(k_{s2} - \frac{1}{4\tau_{s2}}\right)z_{s2}^2 - \frac{1}{\tau_{s2}}(z_{s2}/2 + y_{s2f})^2 + z_{s2}z_{s3} + 0.2785\varsigma_{s2}. \end{aligned}$$

Step 3, ..., n-1,

Following the same design procedure as in *step 2*, it comes to

$$V_{s(i+1)} = V_{si} + \frac{1}{2}z_{si}^2 + \frac{1}{2}y_{si}^2$$

and

$$\dot{V}_{s(n-1)} \leq -k_{s1}z_{s1}^2 - \sum_{i=2}^{n-1} \left(\left(k_{si} - \frac{1}{4\tau_{si}} \right) z_{si}^2 + \frac{1}{\tau_{si}} (z_{si}/2 + y_{si})^2 \right) + z_{s(n-1)}z_{sn} + 0.2785 \sum_{i=2}^{n-1} \varsigma_{si}.$$

Step n,

For identification model I_s , let

$$V_{sn} = V_{s(n-1)} + \frac{1}{2}z_{sn}^2 + \frac{1}{2}y_{sn}^2.$$

As for the proposed multiple model scheme, all the identification models share the same input, it is necessary to take all the identification models into consideration to design the system input.

For the overall system, choose the following Lyapunov function:

$$V = \sum_{s=1}^{q+1} V_{sn}. \tag{8}$$

One can obtain that

$$\begin{aligned} \dot{V} &= \sum_{s=1}^{q+1} \dot{V}_{sn} \\ &\leq - \sum_{s=1}^{q+1} \left\{ k_{s1} z_{s1}^2 + \sum_{i=2}^{n-1} \left[\left(k_{si} - \frac{1}{4\tau_{si}} \right) z_{si}^2 + \frac{1}{\tau_{si}} (z_{si}/2 + y_{sif})^2 \right] \right\} + 0.2785 \sum_{s=1}^{q+1} \sum_{i=2}^{n-1} \zeta_{si} \\ &\quad + \sum_{s=1}^{q+1} \{ z_{s(n-1)} z_{sn} + z_{sn} \dot{z}_{sn} + y_{snf} \dot{y}_{snf} \} \end{aligned}$$

and

$$\begin{aligned} &\sum_{s=1}^{q+1} \{ z_{s(n-1)} z_{sn} + z_{sn} \dot{z}_{sn} + y_{snf} \dot{y}_{snf} \} \\ &= \sum_{s=1}^{q+1} \{ z_{s(n-1)} z_{sn} + z_{sn} (-\lambda (x_{sn} - x_n) + u + \theta_s^T \varphi_n(x, t) - \dot{\alpha}_{s(n-1)}) + y_{snf} \dot{y}_{snf} \} \\ &= \sum_{s=1}^{q+1} \{ z_{s(n-1)} z_{sn} + z_{sn} (-\lambda (x_{sn} - x_n) + u + \theta_s^T \varphi_n(x, t) + k_{sn} z_{sn} - \dot{x}_{snf}) \} \\ &\quad + \sum_{s=1}^{q+1} \{ -k_{sn} z_{sn}^2 + z_{sn} \dot{y}_{snf} + y_{snf} \dot{y}_{snf} \}. \end{aligned}$$

It is clear from *step 2* that

$$\begin{aligned} &\sum_{s=1}^{q+1} \{ -k_{sn} z_{sn}^2 + z_{sn} \dot{y}_{snf} + y_{snf} \dot{y}_{snf} \} \\ &\leq - \sum_{s=1}^{q+1} \left\{ \left(k_{sn} - \frac{1}{4\tau_{sn}} \right) z_{sn}^2 + \frac{1}{\tau_n} (z_{sn}/2 + y_{snf})^2 \right\} + 0.2785 \sum_{s=1}^{q+1} \zeta_{sn}. \end{aligned}$$

For the system input u , the convex combination idea in [19] is used, and it is supposed to have the following form:

$$u = h(x) + \alpha_{*,2} - \sum_{p=1}^{q+1} (\gamma_p \theta_p)^T \varphi_n(x) \tag{9}$$

where $h(x)$ and γ_s are to be designed later and satisfy

$$\begin{cases} \gamma_1 + \gamma_2 + \dots + \gamma_{q+1} = 1 \\ \gamma_s > 0, s = 1, 2, \dots, q + 1 \end{cases} \tag{10}$$

Then,

$$\begin{aligned}
 & \sum_{s=1}^{q+1} \{z_{s(n-1)}z_{sn} + z_{sn} (-\lambda (x_{sn} - x_n) + u + \theta_s^T \varphi_n(x) + k_{sn}z_{sn} - \dot{x}_{snf})\} \\
 &= \sum_{s=1}^{q+1} \left\{ z_{sn} \left(z_{s(n-1)} - \lambda (x_{sn} - x_n) + h(x) + \alpha_{*,2} - \sum_{p=1}^{q+1} (\gamma_p \theta_p)^T \varphi_n(x) + \theta_s^T \varphi_n(x) + k_n z_{sn} \right. \right. \\
 & \quad \left. \left. - \dot{x}_{snf} \right) \right\} \\
 &= \sum_{s=1}^{q+1} z_{sn} (z_{s(n-1)} - \lambda (x_{sn} - x_n) + k_n z_{sn} - \dot{x}_{snf} + h(x)) \\
 & \quad + \sum_{s=1}^{q+1} z_{sn} \left(\alpha_{*,2} + \left(\theta_s - \sum_{p=1}^{q+1} \gamma_p \theta_p \right)^T \varphi_n(x) \right) \\
 & \leq M \cdot \left| \sum_{s=1}^{q+1} z_{sn} \right| + h(x) \sum_{s=1}^{q+1} z_{sn} + N \cdot \left| \sum_{s=1}^{q+1} z_{sn} \right| + \alpha_{*,2} \sum_{s=1}^{q+1} z_{sn}
 \end{aligned}$$

where $M = \max_{1 \leq s \leq q+1} |z_{s(n-1)} - \lambda (x_{sn} - x_n) + k_n z_{sn} - \dot{x}_{snf}|$ and $N = \max_{1 \leq s \leq q+1} |(\theta_s - \sum_{p=1}^{q+1} \gamma_p \theta_p)^T \varphi_n(x)|$. Let

$$\begin{aligned}
 h(x) &= -M \cdot \tanh \left(\frac{M \sum_{s=1}^{q+1} z_{sn}}{\zeta_m} \right) \\
 \alpha_{*,2} &= -N \cdot \tanh \left(\frac{N \sum_{s=1}^{q+1} z_{sn}}{\varepsilon_n} \right).
 \end{aligned} \tag{11}$$

Then,

$$\begin{aligned}
 & \sum_{s=1}^{q+1} \{z_{s(n-1)}z_{sn} + z_{sn} (-\lambda (x_{sn} - x_n) + u + \theta_s^T \varphi_n(x) + k_n z_{sn} - \dot{x}_{snf})\} \\
 & \leq 0.2785\zeta_m + 0.2785\varepsilon_n
 \end{aligned}$$

and

$$\dot{V} \leq - \sum_{s=1}^{q+1} \left\{ k_{s1} z_{s1}^2 + \sum_{i=2}^n \left[\left(k_{si} - \frac{1}{4\tau_{si}} \right) z_{si}^2 + \frac{1}{\tau_{si}} (z_{si}/2 + y_{si_f})^2 \right] \right\} + v \tag{12}$$

where $v = 0.2785 \left(\sum_{s=1}^{q+1} \sum_{i=2}^n \zeta_{si} + \zeta_m + \varepsilon_n \right)$

3.1.2. Second level design. As for the convex index design, set the virtual model with parameter θ_v , which lies in the convex hull of the initial parameters of identification models, then there exist $\{\gamma_1, \gamma_2, \dots, \gamma_{q+1}\}$ such that

$$\begin{cases} \gamma_1 \hat{\theta}_1(t_0) + \gamma_2 \hat{\theta}_2(t_0) + \dots + \gamma_{q+1} \hat{\theta}_{q+1}(t_0) = \theta_v(t_0) \\ \gamma_1 + \gamma_2 + \dots + \gamma_{q+1} = 1 \\ \gamma_i > 0, i = 1, 2, \dots, q+1. \end{cases} \tag{13}$$

Then, one can obtain that

$$\begin{aligned} \dot{\theta}_v(t) &= \sum_{s=1}^{q+1} \gamma_s \dot{\hat{\theta}}_s(t) \\ &= - \sum_{s=1}^{q+1} \gamma_s \sum_{i=1}^n e_{si} \varphi_i(\bar{x}_i) \\ &= - \sum_{i=1}^n \varphi_i(\bar{x}_i) \sum_{s=1}^{q+1} \gamma_s e_{si}. \end{aligned} \tag{14}$$

Further with the error system (3), it follows that the identification error of the virtual model is

$$e_v = \sum_{s=1}^{q+1} \gamma_s e_s \tag{15}$$

and the parameter update is

$$\dot{\theta}_v(t) = - \sum_{i=1}^n e_{vi} \varphi_i(\bar{x}_i) \tag{16}$$

which is identical to all the identification models. This fact further implies that all the virtual models start in the convex hull of initial identification parameters at time t_0 will lie in it at time t , which has been proved in [19].

Then with Lemma 1, it comes that $\gamma_1 e_1 + \gamma_2 e_2 + \dots + \gamma_{q+1} e_{q+1} \rightarrow 0$ exponentially as $t \rightarrow \infty$, which makes it reasonable to set the following equation as in [19]

$$[e_1(t), e_2(t), \dots, e_{q+1}(t)] \Gamma = 0 \tag{17}$$

where $\Gamma = [\gamma_1, \gamma_2, \dots, \gamma_{q+1}]^T$. Also, the character of convex index makes $\Gamma = [\bar{\Gamma}, 1 - \sum_{s=1}^q \gamma_s]$, and (17) can be rewritten as

$$E(t) \bar{\Gamma} = \zeta(t) \tag{18}$$

where $E_s(t) = e_s(t) - e_{q+1}(t)$, $s = 1, 2, \dots, q$ and $\zeta(t) = -e_{q+1}(t)$.

Then the estimate model of (18) can be obtained as

$$E(t) \hat{\bar{\Gamma}} = \zeta(t) \tag{19}$$

and the adaptation law can be easily obtained as

$$\dot{\hat{\bar{\Gamma}}}(t) = -E(t)^T E(t) \hat{\bar{\Gamma}}(t) + E(t)^T \zeta(t), \quad \bar{\Gamma} = Proj_{\hat{\bar{\Gamma}}(t) \in C} \{\hat{\bar{\Gamma}}\} \tag{20}$$

where $C = \{(\gamma_1, \gamma_2, \dots, \gamma_q) \mid \gamma_s > 0, \sum_{s=1}^q \gamma_s < 1, s = 1, 2, \dots, q\}$.

Remark 2

According to Fact 2, $q + 1$ initial points $\hat{\theta}_i(t_0)$ can make a convex hull of unknown parameter θ , which makes sure that $q + 1$ identification models are enough for the proposed multiple model scheme.

Remark 3

When there is no uncertainty for θ , it is possible to use q points to make the convex hull, but it is hard to find such q points, and it cannot cover the uncertainty case. Also, $q + m$ ($m > 1$) points can be used to make the convex hull. However, the convex combination indexes are not unique any more, which leads to more difficulties in analysis.

3.2. Fixed models

When all the identification models are adaptive, it is possible that the parameters of each identification model converge to a settle point or a small region of the system parameter. If the system parameter changes, all the identification models start from one point. Then, there is nothing but a single model adaptive control. So, the fixed models in multiple model scheme are used here. The parameter of identification models is fixed, and the convex indexes will be adaptively updated.

Suppose the system parameter $\theta = \sum_{s=1}^{q+1} \gamma_s \theta_s$, then the system model can be written as

$$\begin{cases} \dot{x}_i = x_{i+1} + \left(\sum_{s=1}^{q+1} \gamma_s \theta_s\right)^T \varphi_i(\bar{x}_i), & i = 1, 2, \dots, n-1 \\ \dot{x}_n = u + \left(\sum_{s=1}^{q+1} \gamma_s \theta_s\right)^T \varphi_n(x) \\ y = x_1. \end{cases} \quad (21)$$

Define $\bar{\theta} = [\theta_1, \theta_2, \dots, \theta_{q+1}]$, $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_{q+1}]^T$ and $\psi_i(\bar{x}_i) = \bar{\theta}^T * \varphi_i(\bar{x}_i)$, then

$$\begin{cases} \dot{x}_i = x_{i+1} + \gamma^T \psi_i(\bar{x}_i) & i = 1, 2, \dots, n-1 \\ \dot{x}_n = u + \gamma^T \psi_n(x) \\ y = x_1 \end{cases} \quad (22)$$

where γ is treated as an unknown parameter that satisfies (10). By using this transformation, the uncertainty of θ is replaced by γ , which is in a unit ball, and the uncertainty is much smaller. Then by applying the design methods in [4][21], it comes that all the signals in (22) are bounded, and the system output uniformly tends to an arbitrary small bound of the original point.

4. STABILITY ANALYSIS

In this section, the stability analysis is given for the proposed multiple model scheme. When the identification models are adaptive, the stability analysis is based on the controller design procedure in Section 3.1 and presented in Theorem 1. When the identification models are fixed, the controller design and the stability analysis are similar to [21] and presented in Theorem 2.

Theorem 1

Consider the system (1), identification model (2), parameter adaptation law (6), and controller (9)-(11), then there exist k_{si}, τ_{si} such that all the signals are uniformly bounded and z_{si}, y_{si_f} can converge to an arbitrary small neighborhood of the original point.

Proof

From Lyapunov function (8), it can be obtained that

$$\dot{V} \leq - \sum_{s=1}^{q+1} \left\{ k_{s1} z_{s1}^2 + \sum_{i=2}^n \left[\left(k_{si} - \frac{1}{4\tau_{si}} \right) z_{si}^2 + \frac{1}{\tau_{si}} (z_{si}/2 + y_{si_f})^2 \right] \right\} + v \quad (23)$$

where $v = 0.2785(\sum_{s=1}^{q+1} \sum_{i=2}^n \zeta_{si} + \zeta_m + \varepsilon_n)$ if there exist the following control parameters:

$$\begin{cases} k_{s1} > 0 \\ k_{si} - \frac{1}{4\tau_{si}} > 0 \\ \tau_{si} > 0 \\ s = 1, 2, \dots, q+1 \\ i = 1, 2, \dots, n \end{cases} \quad (24)$$

then,

$$\dot{V} \leq -2\eta V + v$$

where η is a designed parameter. As ν is also a designed parameter, then using the standard Lyapunov analysis, it comes to the conclusion that all the signals in the identification models are uniformly bounded, and z_{si}, y_{si_f} can converge to an arbitrary small neighborhood of the original point.

Then, according to Lemma 2, the signals of system (1) are bounded, and the output tends an arbitrary small neighborhood of the original point uniformly. \square

Remark 4

According to Remark 1 and the character of the convex indexes, the conclusion can be drawn that under the condition of Remark 1, the combination of parameter estimate converges to the true parameter.

Theorem 2

Consider the system (1) and (22) for fixed identification model set. If the controller design and parameter adaptation law in [4][21] are applied, asymptotical output tracking and true parameter estimates can be achieved if proper control parameters are carefully designed.

Proof

Consider the original system (1), if the transformation (21) is applied, it can be rewritten as (22), which is in canonical strict-feedback form with γ as the unknown parameter. Then by applying the design procedure in [4][21], the conclusion can be drawn that the tracking error can converge to a designed small region of zero asymptotically. \square

Remark 5

By applying the transformation (21), the uncertainty zone of (22) is just a unit ball, which may be much smaller than the uncertainty of the original system (1), as the multiple model scheme is mainly proposed for the system with large uncertainty. Then, it is much easier and more efficient to design the controller.

5. SIMULATION EXAMPLES

To illustrate the advantage of the proposed multiple model scheme, two simulation examples are given in this section.

Example 1

Consider the following nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 + \theta \cdot \varphi_1(x_1) \\ \dot{x}_2 = u + \theta \cdot \varphi_2(x_1, x_2) \\ y = x_1 \end{cases} \quad (25)$$

where $\varphi_1(x_1) = \cos(x_1)$, $\varphi_2(x_1, x_2) = x_1 + \cos(x_2)$, and $\theta \in [1, 20]$ is the unknown parameter. Suppose the true parameter $\theta^* = 4$. The initial parameter for the classical adaptive controller is $\theta(0) = 10$. For multiple model with switching, five identification models are used with initial parameters as $\{1, 5, 10, 15, 20\}$. For multiple model with convex index, two identification models are used with initial parameters $\{1, 20\}$ and initial convex indexes $\gamma_1 = 0.3$, $\gamma_2 = 0.7$. The objective is to regulate the output to 0, and the simulation results are shown in Figures 1–3.

As shown in Figure 1, the system output of different schemes all tend to zero uniformly but with different transient performance. According to the solid line, the system needs a long time to settle down when using the classical adaptive control, and the transient performance is not very good with a big overshoot. Then for the multiple model with switching scheme, the settling time is almost the same, but the transient performance improves a lot, where the overshoot is much smaller. It follows from the point-dash line that when the smooth controller with the convex multiple model scheme is

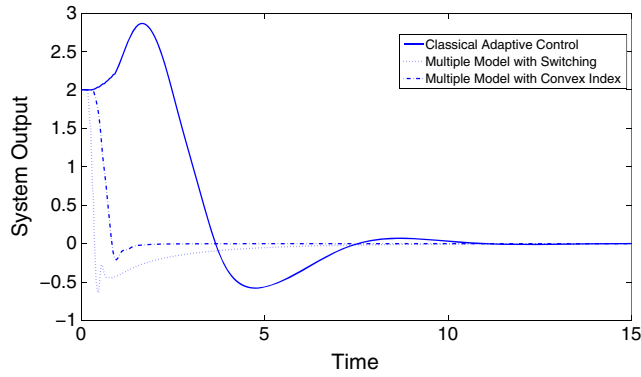


Figure 1. The system output.

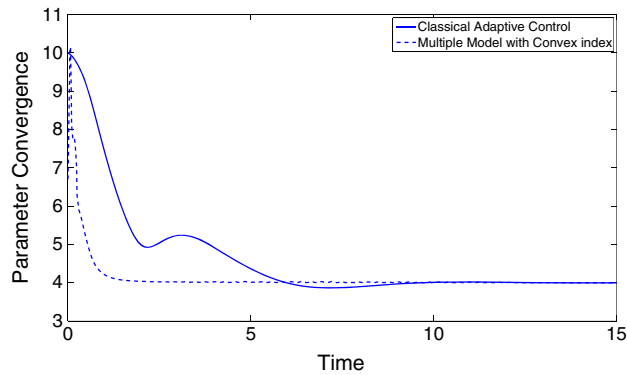


Figure 2. The parameter convergence.

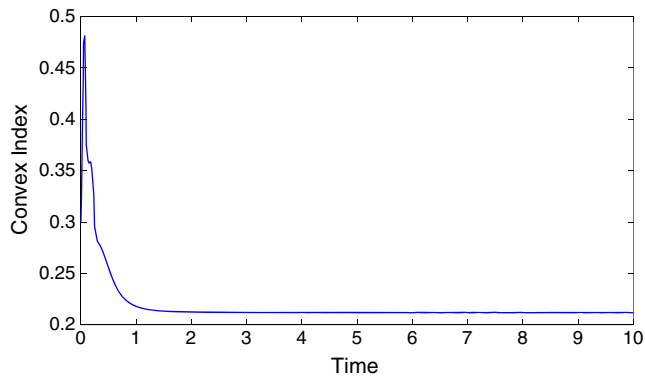


Figure 3. The convex index.

applied, the performance is more smooth and the settling time also decreases, which demonstrates the efficiency of the proposed method. For the parameter convergence in Figure 2, both of the classical adaptive method and the proposed method can converge to the true value, and the proposed scheme possesses a deeply improved transient performance. In Figure 3, it follows that the convex index converges to a steady value, which is proved in the theory, and also corresponds to the convergence of the parameter.

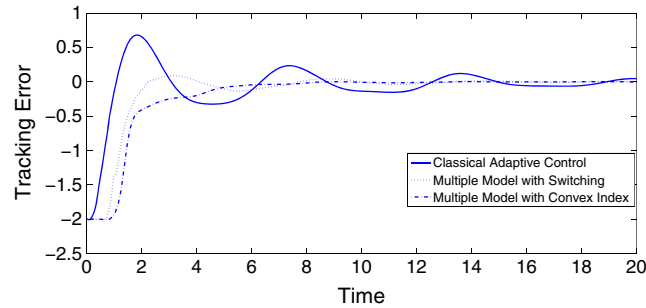


Figure 4. The tracking error.

Example 2

Consider the following nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 + \theta^T \cdot \varphi_1(x_1) \\ \dot{x}_2 = u + \theta^T \cdot \varphi_2(x_1, x_2) \\ y = x_1 \end{cases} \quad (26)$$

where $\varphi_1(x_1) = [\cos(x_1), x_1 + \cos(x_1)]^T$, $\varphi_2(x_1, x_2) = [\cos(x_1), x_1 + \cos(x_2)]^T$, $\theta = [\theta_1, \theta_2]^T$, $\theta_1 \in [0, 10]$, $\theta_2 \in [0, 10]$ is the unknown parameter. Suppose the true parameter $\theta^* = [3, 2]^T$. The initial parameter for the classical adaptive controller is $\theta(0) = [6, 6]^T$. For multiple model with switching, 10 identification models are used with initial parameters as

$$\{[0, 0]^T, [0, 5]^T, [0, 10]^T, [3, 0]^T, [3, 5]^T, [3, 10]^T, [6, 0]^T, [6, 5]^T, [6, 10]^T, [9, 2]^T\}.$$

For multiple model with convex index, three identification models are used with initial parameters

$$\{[0, 0]^T, [0, 10]^T, [10, 0]^T\}$$

and initial convex index $\gamma_1 = 0.3$, $\gamma_2 = 0.3$, $\gamma_3 = 0.4$. The objective is to track the signal $\sin(t)$.

As shown in Figure 4, all of the output under different methods can track the signal $\sin(t)$ very well. However, the classical adaptive control needs more time to track the signal than the others, and the transient performance is not very good. For the multiple model scheme, both of them can track the signal in a shorter time, and the tracking error is much smaller than the classical method. Further, the proposed method has a smoother performance than the switching scheme.

6. CONCLUSION

This paper has presented a new multiple model scheme with convex index for strict-feedback nonlinear systems. A continuous controller with convex combination of the parameter estimates of the identification models has been used to avoid the possible chattering and to improve the transient performance. Because of the convex hull property, only $q + 1$ models are needed, which is more practical than the switching scheme. Also, the information of identification models has been more fully used. The simulation examples have demonstrated the efficiency of the proposed scheme.

ACKNOWLEDGEMENTS

This work was supported in part by the Natural Science Foundation of China under Grant 61104097, the National Science Foundation for Distinguished Young Scholars of China under Grant 60925011, the Projects of Major International (Regional) Joint Research Program Natural Science Foundation of China under Grant 61120106010, the Beijing Education Committee Cooperation Building Foundation Project XK100070532, and the Research Fund for the Doctoral Program of Higher Education of China 20111101120027.

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