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Distributed tracking for networked Euler-Lagrange systems without velocity measurements

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Abstract: The problem of distributed coordinated tracking control for networked Euler-Lagrange systems without velocity measurements is investigated in this paper. Under the condition that only a portion of the followers have access to the leader, sliding mode estimators are developed to estimate the states of the dynamic leader in finite time. To cope with the absence of velocity measurements, the distributed observers which only use position information are designed. Based on the outputs of the estimators and observers, distributed tracking control laws are proposed such that all the followers with parameter uncertainties can track the dynamic leader under a directed graph containing a spanning tree. It is shown that the distributed observer-controller guarantees asymptotical stability of the closed-loop system. Numerical simulations are worked out to illustrate the effectiveness of the control laws.

Keywords: Euler-Lagrange system, distributed control, coordinated tracking, velocity observer.

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1. Introduction

Distributed control of multi-agent systems has received increasing concern due to its broad applications in various fields, ranging from physics, engineering to biology [1,2]. Currently, most research works on multi-agent systems mainly concentrate on linear systems [3–5]. However, for some kinds of mechanical systems, such as autonomous vehicles, robotic manipulators and rigid bodies, the nonlinear dynamics can not be neglected in practice. Hence it is of great significance to study coordinated control of Euler-Lagrange systems which can generally describe motion of mechanical systems [6,7]. The problem of distributed control for networked Euler-Lagrange systems recently has been studied in [8–14]. The consensus of multiple Euler-Lagrange systems with self-delays and uncertainties under strongly connected digraph was considered in [8]. In [9,10], the problem of leaderless consensus for multiple Euler-Lagrange systems was investigated. The adaptive consensus protocols in the case of time-delay and switching topology with the assumption that the graph was balanced were proposed in [9]. The output synchronization of the multiple Euler-Lagrange systems was achieved under both fixed and switching topology based on the passivity property of mechanical systems in [10]. The problem of tracking a leader or equivalently a reference for a class of mechanical systems modeled by Euler-lagrange equations was studied in [11–14]. In [11], the nonlinear contraction theory was applied to prove the stability for multiple robotic manipulators. However, it was required that all the followers should have access to the dynamic leader, which is rather restrictive. A distributed adaptive control law was designed to track the reference trajectory for multiple uncertain mechanical systems in [12], where the topology was required to be undirected. Reference [13] coped with the tracking problem for networked Euler-Lagrange systems under the condition that the leader's generalized coordinate derivative was constant and time-varying, respectively. Accordingly, an adaptive control law together with a distributed continuous estimator in the case of stationary leader and a model-independent sliding mode control algorithm in the case of dynamic leader were proposed. In [14], the containment problem (i.e., tracking with multiple leaders) was concerned in the presence of parametric uncertainties under a directed graph. Similarly, a distributed adaptive controller was proposed such that the followers could converge into the convex hull spanned by leaders. Although the distributed coordinated control was

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realized on directed graphs in [13,14], the full state information was required in the proposed controllers.

It should be noticed that all the above mentioned research works are conducted under the condition that all the followers' information is required to be available in order to implement the controller design. Nevertheless, in practice, velocity and acceleration measurements may not always be measurable due to the strict constraints on the cost and space for installing the devices. In [15], the containment control algorithms via only position measurements were proposed based on the super twisting algorithm. The coordinated control problem without velocity measurements was studied in [16,17]. However, [15–17] only considered the systems with linear integrator models. Up to now, there are very few papers dealing with distributed control of nonlinear systems without velocity measurements. The distributed tracking problem with a dynamic leader using only position measurements was investigated in [18]. Ren proposed distributed algorithms for networked Euler-Lagrange systems without velocity information [19]. However, the undirected topology among followers should be connected in [18,19].

In this paper, only the position measurements among the followers are used to achieve distributed coordinated tracking for networked Euler-Lagrange systems with parameter uncertainties, while moreover only a subset of followers have access to the leader on a directed graph. To overcome this challenging problem, new observers and estimators are designed with the aid of followers' position measurements, then distributed control laws are proposed based on the outputs of the observers and estimators. The main contributions of this paper are that a new framework of distributed observers and an observer-based adaptive control method are proposed, which are based on a novel term that is elaborately designed to compensate the absence of velocity measurements. Besides, the stability of the system is proved theoretically based on algebraic graph theory and Lyapunov analysis.

The subsequent sections are organized as follows. Section 2 introduces the model of the agents to be controlled and the graph theory. In Section 3, distributed sliding-mode estimators are developed in the first part. Then, distributed velocity observers are presented, followed by the main result (Theorem 1) and the stability analysis. Numerical examples are carried out to show the effectiveness of the proposed control algorithms in Section 4. Section 5 gives the conclusion.

2. Background

A team of n mechanical systems labeled as agents 1 to n are considered as followers, and a leader is labeled as agent 0. They are all described by Euler-Lagrange equations as follows:

$$\boldsymbol{M}_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i + \boldsymbol{G}_i(\boldsymbol{q}_i) = \boldsymbol{\tau}_i, \quad i = 1, \cdots, n \tag{1}$$

where $\boldsymbol{q}_i \in \mathbf{R}^p$ is the vector of generalized coordinates, $\boldsymbol{M}_i(\boldsymbol{q}_i) \in \mathbf{R}^{p\times p}$ is the symmetric positive-definite inertia matrix, $\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i \in \mathbf{R}^p$ is the vector of Coriolis and centrifugal torques, $\boldsymbol{G}_i(\boldsymbol{q}_i)$ is the vector of gravitational torques, and $\boldsymbol{\tau}_i \in \mathbf{R}^p$ is the vector of control torque on the agent i.

In the following, we define, respectively, $\boldsymbol{M}(\boldsymbol{q}) \triangleq \mathrm{diag}[\boldsymbol{M}_1(\boldsymbol{q}_1), \dots, \boldsymbol{M}_n(\boldsymbol{q}_n)]$, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \triangleq \mathrm{diag}[\boldsymbol{C}_1(\boldsymbol{q}_1, \dot{\boldsymbol{q}}_1), \cdots, \boldsymbol{C}_n(\boldsymbol{q}_n, \dot{\boldsymbol{q}}_n)]$, and $\boldsymbol{G}(\boldsymbol{q}) \triangleq [\boldsymbol{G}_1^{\mathrm{T}}(\boldsymbol{q}_1), \cdots, \boldsymbol{G}_n^{\mathrm{T}}(\boldsymbol{q}_n)]^{\mathrm{T}}$. Let $\|\cdot\|$ and $\boldsymbol{x}^{(i)}$ denote the Euclidean norm and the ith order derivative of $\boldsymbol{x}$, respectively. For (1), the following properties hold [20,21].

Property 1 For any i, there exist positive constants $k_{\underline{m}}$, $k_{\overline{m}}$ and k_C such that $0 < k_{\underline{m}}\boldsymbol{I}_p \leqslant \boldsymbol{M}_i(\boldsymbol{q}_i) \leqslant k_{\overline{m}}\boldsymbol{I}_p$, $\|\boldsymbol{C}_i(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{z}\| \leqslant k_C\|\boldsymbol{y}\|\|\boldsymbol{z}\|$ for all vectors $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \mathbf{R}^p$.

Property 2 $\dot{\boldsymbol{M}}_i(\boldsymbol{q}_i) - 2\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$ is skew symmetric:

$$\boldsymbol{\xi}^{\mathrm{T}}[\dot{\boldsymbol{M}}_i(\boldsymbol{q}_i) - 2\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)]\boldsymbol{\xi} = 0, \quad \forall \boldsymbol{\xi} \in \mathbf{R}^p. \tag{2}$$

Property 3 Equation (1) is linearly parameterizable:

$$\boldsymbol{M}_i(\boldsymbol{q}_i)\boldsymbol{\xi} + \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\boldsymbol{\zeta} + \boldsymbol{G}_i(\boldsymbol{q}_i) = \boldsymbol{Y}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i, \boldsymbol{\xi}, \boldsymbol{\zeta})\boldsymbol{\Theta}_i \tag{3}$$

for all vectors $\boldsymbol{\xi}, \boldsymbol{\zeta} \in \mathbf{R}^p$, where $\boldsymbol{Y}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i, \boldsymbol{\xi}, \boldsymbol{\zeta})$ is the regressor and $\boldsymbol{\Theta}_i$ is an unknown but constant parameter vector associated with the agent i.

$\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ is used to represent the interactions among the agents 1 to n with the node set $\mathcal{V} \triangleq \{1, \dots, n\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The edge (i, j) denotes that the agent i transmits information to the agent j in a directed graph, but not vice versa. In an undirected graph, an edge $(i, j) \in \mathcal{E}$ if agents i and j can receive information from each other. Here, it is assumed that there is no loop in the graph, i.e., $(i, i) \notin \mathcal{E}$. If an edge $(i, j) \in \mathcal{E}$, then we call the node i a neighbor of the node j. Thus, the neighbor set of the agent i is defined as $\mathcal{N}_i \triangleq \{j|(j, i) \in \mathcal{E}\}$. The root is a node that has directed paths to all the other nodes in a directed graph. A directed tree contains exactly one root and every other node has only one parent. A directed tree is called a directed spanning tree if it consists of all the nodes in a graph. A directed graph contains a directed spanning tree as long as one of its subgraphs is a directed spanning tree [22]. The adjacency matrix $\boldsymbol{A} = [a_{ij}] \in \mathbf{R}^{n\times n}$ is defined such that $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Note that, $a_{ij} = a_{ji}$ in an undirected graph. Let the Laplacian matrix $\boldsymbol{L} = [l_{ij}] \in \mathbf{R}^{n\times n}$, with $l_{ii} = \sum\limits_{j=1, j\neq i}^{n} a_{ij}$

and $l_{ij} = -a_{ij}, i \neq j$.

Lemma 1 [23] Let $\overline{\mathcal{G}} \triangleq (\overline{\mathcal{V}}, \overline{\mathcal{E}})$ be the directed graph characterizing the interaction among the n followers and the leader, accordingly, $a_{i0} > 0$ if $(0, i) \in \mathcal{E}$ and $a_{i0} = 0$ otherwise. Considering the extended graph $\overline{\mathcal{G}}$, the generalized Laplacian matrix $\boldsymbol{H} \triangleq \boldsymbol{L} + \mathrm{diag}(a_{10}, \ldots, a_{n0})$ is positive stable if the leader has directed paths to all the followers.

Assumption 1 The desired trajectory $\boldsymbol{q}_0$ is differentiable, and its derivatives are bounded, i.e., $\|\dot{\boldsymbol{q}}_0\|_\infty \leqslant \eta_1, \|\ddot{\boldsymbol{q}}_0\|_\infty \leqslant \eta_2$ and $\|\boldsymbol{q}_0^{(3)}\|_\infty \leqslant \eta_3$, where $\eta_i (i = 1, 2, 3)$ are positive scalars.

Assumption 2 The initial observation and tracking errors of the ith agent are bounded, i.e., $\boldsymbol{e}_{oi}^{(j)}(0) \leqslant \delta_0$ and $\tilde{\boldsymbol{e}}_i^{(j)}(0) \leqslant \delta_1$ $(i = 1, 2, \ldots, n, j = 0, 1, 2)$, where $\boldsymbol{e}_{oi}$ and $\tilde{\boldsymbol{e}}_i$ are defined later.

3. Main results

To deal with the problem of distributed tracking control without velocity measurements, new distributed tracking control algorithms are proposed for the networked Euler-Lagrange systems under the interaction topology where the leader is only available to a subset of the followers.

3.1 Estimator design

Since the states of the leader are only known by parts of the followers, inspired by [14], the distributed sliding-mode estimators are designed to estimate the information of the leader as

$$\dot{\tilde{\boldsymbol{q}}}_{0i} = -\alpha_1 \mathrm{sgn}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\tilde{\boldsymbol{q}}_{0i} - \tilde{\boldsymbol{q}}_{0j}) + a_{i0}(\tilde{\boldsymbol{q}}_{0i} - \boldsymbol{q}_0)\right] \tag{4a}$$

$$\dot{\tilde{\boldsymbol{v}}}_{0i} = -\alpha_2 \mathrm{sgn}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\tilde{\boldsymbol{v}}_{0i} - \tilde{\boldsymbol{v}}_{0j}) + a_{i0}(\tilde{\boldsymbol{v}}_{0i} - \dot{\boldsymbol{q}}_0)\right] \tag{4b}$$

$$\dot{\tilde{\boldsymbol{a}}}_{0i} = -\alpha_3 \mathrm{sgn}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\tilde{\boldsymbol{a}}_{0i} - \tilde{\boldsymbol{a}}_{0j}) + a_{i0}(\tilde{\boldsymbol{a}}_{0i} - \ddot{\boldsymbol{q}}_0)\right] \tag{4c}$$

where $\tilde{\boldsymbol{q}}_{0i}$ ($\tilde{\boldsymbol{v}}_{0i}$ and $\tilde{\boldsymbol{a}}_{0i}$, respectively) is the ith follower's estimation of the leader's generalized coordinate (derivative and acceleration, respectively), which is denoted by $\boldsymbol{q}_0$, ($\dot{\boldsymbol{q}}_0$ and $\ddot{\boldsymbol{q}}_0$, respectively). a_{ij} $(i, j = 1, \ldots, n)$ is the entry of the adjacency matrix. α_1, α_2 and α_3 are positive constants, and sgn$(\cdot)$ is the signum function accordingly.

Lemma 2 [14,24] Suppose that in $\overline{\mathcal{G}}$, the leader has directed paths to all the followers. Then, $\|\tilde{\boldsymbol{q}}_{0i} - \boldsymbol{q}_0\| \to 0$ in finite time, if $\alpha_1 > \|\dot{\boldsymbol{q}}_0\|$, and the upper bound of the settling time is denoted by T_1. Similarly, $\|\tilde{\boldsymbol{v}}_{0i} - \dot{\boldsymbol{q}}_0\| \to 0$ in finite time is denoted by T_2, if $\alpha_2 > \|\ddot{\boldsymbol{q}}_0\|$, and $\|\tilde{\boldsymbol{a}}_{0i} - \ddot{\boldsymbol{q}}_0\| \to 0$ in finite time denoted by T_3, if $\alpha_3 > \|\boldsymbol{q}_0^{(3)}\|, i = 1, \ldots, n$.

Remark 1 The results in [14] can be extended to the case in this paper, and the settling time would be $T_1 = \dfrac{\max\|\bar{\boldsymbol{q}}_{0i}(0)\|_\infty}{\alpha_1 - \sup\limits_{t\geqslant 0}\|\dot{\boldsymbol{q}}_0\|}$, $T_2 = \dfrac{\max\|\bar{\boldsymbol{v}}_{0i}(0)\|_\infty}{\alpha_2 - \sup\limits_{t\geqslant 0}\|\ddot{\boldsymbol{q}}_0\|}$, and $T_3 = \dfrac{\max\|\bar{\boldsymbol{a}}_{0i}(0)\|_\infty}{\alpha_3 - \sup\limits_{t\geqslant 0}\|\boldsymbol{q}_0^{(3)}\|}$, where $\bar{\boldsymbol{q}}_{0i} = \tilde{\boldsymbol{q}}_{0i} - \boldsymbol{q}_0$, $\bar{\boldsymbol{v}}_{0i} = \tilde{\boldsymbol{v}}_{0i} - \dot{\boldsymbol{q}}_0$, and $\bar{\boldsymbol{a}}_{0i} = \tilde{\boldsymbol{a}}_{0i} - \ddot{\boldsymbol{q}}_0$, respectively.

3.2 Observer-controller design

Define the following auxiliary variables:

$$\dot{\boldsymbol{q}}_{ri} \triangleq \tilde{\boldsymbol{v}}_{0i} - \beta_1\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{q}_i - \boldsymbol{q}_j) + a_{i0}(\boldsymbol{q}_i - \boldsymbol{q}_0)\right] \tag{5}$$

$$\tilde{\boldsymbol{s}}_i \triangleq \dot{\boldsymbol{q}}_i - \dot{\boldsymbol{q}}_{ri} = \tilde{\boldsymbol{e}}_i + \beta_1\Big[\sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{q}_i - \boldsymbol{q}_j) + a_{i0}(\boldsymbol{q}_i - \boldsymbol{q}_0)\Big] \tag{6}$$

$$\boldsymbol{s}_{oi} \triangleq \dot{\boldsymbol{e}}_{oi} + \beta_2\sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{e}_{oi} - \boldsymbol{e}_{oj}), \quad i = 1, \ldots, n \tag{7}$$

where β_1 and β_2 are positive constants, $\tilde{\boldsymbol{e}}_i \triangleq \dot{\boldsymbol{q}}_i - \tilde{\boldsymbol{v}}_{0i}$, $\boldsymbol{e}_{oi} \triangleq \boldsymbol{q}_i - \hat{\boldsymbol{q}}_i$, and $\dot{\boldsymbol{e}}_{oi} \triangleq \dot{\boldsymbol{q}}_i - \hat{\dot{\boldsymbol{q}}}_i$. Here $\hat{\boldsymbol{q}}_i$ ($\hat{\dot{\boldsymbol{q}}}_i$, respectively) is the observation of the generalized coordinate (derivative, respectively) of the agent i, a_{ij} $(i, j = 1, \ldots, n)$ is the entry of the adjacency matrix. Here, (6) and (7) can be written in vector forms as

$$\tilde{\boldsymbol{s}} \triangleq \tilde{\boldsymbol{e}} + \beta_1(\boldsymbol{H} \otimes \boldsymbol{I}_p)\boldsymbol{e} \tag{8}$$

$$\boldsymbol{s}_o \triangleq \dot{\boldsymbol{e}}_o + \beta_2(\boldsymbol{L} \otimes \boldsymbol{I}_p)\boldsymbol{e}_o \tag{9}$$

where $\boldsymbol{e} \triangleq \boldsymbol{q} - \boldsymbol{q}_0$, $\boldsymbol{e}$ is defined as the position tracking error relative to the leader, $\boldsymbol{e}_o$ is the vector form of the observation error, and $\tilde{\boldsymbol{e}}$ denotes the velocity errors between the followers and the estimations of the leader.

Applying Property 3 to the term $\boldsymbol{M}_i(\tilde{\boldsymbol{q}}_{0i})\tilde{\boldsymbol{a}}_{0i} + \boldsymbol{C}_i(\tilde{\boldsymbol{q}}_{0i}, \tilde{\boldsymbol{v}}_{0i})\tilde{\boldsymbol{v}}_{0i} + \boldsymbol{G}_i(\tilde{\boldsymbol{q}}_{0i})$ and combining system model (1), we have

$$\boldsymbol{M}_i(\boldsymbol{q}_i)\dot{\tilde{\boldsymbol{s}}}_i + \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\tilde{\boldsymbol{s}}_i + \Delta\tilde{\boldsymbol{W}}_i = \boldsymbol{\tau}_i - \tilde{\boldsymbol{\tau}}_{0i} \tag{10}$$

where

$$\Delta\tilde{\boldsymbol{W}}_i = \boldsymbol{M}_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_{ri} + \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_{ri} + \boldsymbol{G}_i(\boldsymbol{q}_i) - \boldsymbol{Y}_i(\tilde{\boldsymbol{q}}_{0i}, \tilde{\boldsymbol{v}}_{0i}, \tilde{\boldsymbol{a}}_{0i})\boldsymbol{\Theta}_i$$

$$\tilde{\boldsymbol{\tau}}_{0i} = \boldsymbol{M}_i(\tilde{\boldsymbol{q}}_{0i})\tilde{\boldsymbol{a}}_{0i} + \boldsymbol{C}_i(\tilde{\boldsymbol{q}}_{0i}, \tilde{\boldsymbol{v}}_{0i})\tilde{\boldsymbol{v}}_{0i} + \boldsymbol{G}_i(\tilde{\boldsymbol{q}}_{0i}) = \boldsymbol{Y}_i(\tilde{\boldsymbol{q}}_{0i}, \tilde{\boldsymbol{v}}_{0i}, \tilde{\boldsymbol{a}}_{0i})\boldsymbol{\Theta}_i$$

The vector form of (10) is

$$\boldsymbol{M}(\boldsymbol{q})\dot{\tilde{\boldsymbol{s}}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\tilde{\boldsymbol{s}}+\Delta\tilde{\boldsymbol{W}}=\boldsymbol{\tau}-\tilde{\boldsymbol{\tau}}_0. \tag{11}$$

Lemma 3 Define $\boldsymbol{s}\triangleq\dot{\boldsymbol{e}}+\beta_1(\boldsymbol{H}\otimes\boldsymbol{I}_p)\boldsymbol{e}$, where $\dot{\boldsymbol{e}}=\dot{\boldsymbol{q}}-\dot{\boldsymbol{q}}_0$, and $\Delta\boldsymbol{W}\triangleq\boldsymbol{M}(\boldsymbol{q})(\ddot{\boldsymbol{q}}-\dot{\boldsymbol{s}})+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})(\dot{\boldsymbol{q}}-\boldsymbol{s})+\boldsymbol{G}(\boldsymbol{q})-\boldsymbol{Y}(\boldsymbol{q}_0,\dot{\boldsymbol{q}}_0,\ddot{\boldsymbol{q}}_0)\boldsymbol{\Theta}$, there exist positive definite functions b_{11}, b'_{12}, b''_{12}, b'_{13}, $b_{21}, b'_{21}, b''_{22}, b'_{23}$ such that the following inequalities hold,

$$\begin{aligned}-\boldsymbol{s}^{\mathrm{T}}\Delta\boldsymbol{W}\leqslant{}&\beta_1\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{H}\boldsymbol{s}-\beta_1^2\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{e}+\\&b_{11}\|\boldsymbol{s}\|^2+b_{12}\|\boldsymbol{s}\|\|\boldsymbol{e}\|+b_{13}(\|\boldsymbol{s}\|^2\|\boldsymbol{e}\|+\\&\beta_1\overline{\sigma}(\boldsymbol{H})\|\boldsymbol{s}\|\|\boldsymbol{e}\|^2)\end{aligned} \tag{12}$$

$$\begin{aligned}-\boldsymbol{s}_o^{\mathrm{T}}\Delta\boldsymbol{W}\leqslant{}&\beta_2\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{L}\boldsymbol{s}-\beta_2^2\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{e}+\\&b_{21}\|\boldsymbol{s}\|\|\boldsymbol{s}_o\|+b_{22}\|\boldsymbol{s}_o\|\|\boldsymbol{e}\|+b_{23}(\|\boldsymbol{s}\|\|\boldsymbol{s}_o\|\|\boldsymbol{e}\|+\\&\beta_2\overline{\sigma}(\boldsymbol{L})\|\boldsymbol{s}_o\|\|\boldsymbol{e}\|^2)\end{aligned} \tag{13}$$

where $b_{12}=b'_{12}+\beta_1\overline{\sigma}(\boldsymbol{H})b''_{12}$, $b_{13}=\beta_1\overline{\sigma}(\boldsymbol{H})b'_{13}$, $b_{22}=b'_{22}+\beta_2\overline{\sigma}(\boldsymbol{L})b''_{22}$, and $b_{23}=\beta_2\overline{\sigma}(\boldsymbol{L})b'_{23}$, $\overline{\sigma}(\boldsymbol{X})$ denotes the maximum singular value of the matrix $\boldsymbol{X}$. Please refer to [25] for the details of the proof of Lemma 3.

To cope with the problem that the velocity of the follower is unavailable, the distributed velocity observers are proposed as follows, in which only the local information such as its own on-board measurements and communicating data from directly connected neighbors, is used.

$$\begin{cases}\hat{\boldsymbol{q}}_i=\boldsymbol{z}_i-\beta_1\left[\sum\limits_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{q}_i-\boldsymbol{q}_j)-a_{i0}(\boldsymbol{q}_i-\boldsymbol{q}_0)\right]+\\\qquad\beta_2\sum\limits_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{e}_{oi}-\boldsymbol{e}_{oj})+k_3\boldsymbol{e}_{oi}\\\dot{\boldsymbol{z}}_i=\tilde{\boldsymbol{a}}_{0i}-k_2(\tilde{\boldsymbol{s}}_i-\boldsymbol{s}_{oi})+k_3\beta_2\sum\limits_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{e}_{oi}-\boldsymbol{e}_{oj})\end{cases} \tag{14}$$

where the gains k_2 and k_3 are positive constants. Note that the term $\tilde{\boldsymbol{s}}_i-\boldsymbol{s}_{0i}=\hat{\boldsymbol{q}}_i-\tilde{\boldsymbol{v}}_{0i}+\beta_1\left[\sum\limits_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{q}_i-\boldsymbol{q}_j)+a_{i0}(\boldsymbol{q}_i-\boldsymbol{q}_0)\right]-\beta_2\sum\limits_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{e}_{oi}-\boldsymbol{e}_{oj})$, therefore the term $\tilde{\boldsymbol{s}}_i-\boldsymbol{s}_{oi}$ does not include any information of the derivatives of the followers' generalized coordinate [26]. Moreover, considering the fact that only the local information is used in the observers, the velocity observation can be realized in a distributed manner, which has significant meaning for cooperative tracking under realistic working conditions. Then, (14) can be written in a vector form as

$$\begin{cases}\hat{\boldsymbol{q}}=\boldsymbol{z}-\beta_1(\boldsymbol{H}\otimes\boldsymbol{I}_p)\boldsymbol{e}+\beta_2(\boldsymbol{L}\otimes\boldsymbol{I}_p)\boldsymbol{e}_o+k_3\boldsymbol{e}_o\\\dot{\boldsymbol{z}}=\tilde{\boldsymbol{a}}_0-k_2(\tilde{\boldsymbol{s}}-\boldsymbol{s}_o)+k_3\beta_2(\boldsymbol{L}\otimes\boldsymbol{I}_p)\boldsymbol{e}_o\end{cases} \tag{15}$$

where $\hat{\boldsymbol{q}},\boldsymbol{z},\boldsymbol{e}_o,\tilde{\boldsymbol{a}}_0,\tilde{\boldsymbol{s}}$ and $\boldsymbol{s}_o$ are, respectively, the column stack vectors of $\hat{\boldsymbol{q}}_i,\boldsymbol{z}_i,\boldsymbol{e}_{oi},\tilde{\boldsymbol{a}}_{0i},\tilde{\boldsymbol{s}}_i$ and $\boldsymbol{s}_{oi}$ $(i=1,2,\dots,n)$.

Based on the outputs of the estimators and velocity observers, here we use $\boldsymbol{Y}_{i0}$ to denote $\boldsymbol{Y}_i(\tilde{\boldsymbol{q}}_{0i},\tilde{\boldsymbol{v}}_{0i},\tilde{\boldsymbol{a}}_{0i})$ and the distributed control law for the agent i can be designed as

$$\boldsymbol{\tau}_i=-\boldsymbol{K}_{1i}(\tilde{\boldsymbol{s}}_i-\boldsymbol{s}_{oi})+\boldsymbol{Y}_{i0}\hat{\boldsymbol{\Theta}}_i-2\boldsymbol{\Lambda}_i\boldsymbol{Y}_{i0}\boldsymbol{Y}_{i0}^{\mathrm{T}}\tilde{\boldsymbol{e}}_i, \tag{16}$$

where $\boldsymbol{K}_{1i}$ and $\boldsymbol{\Lambda}_i$ are symmetric positive-definite matrices, $\tilde{\boldsymbol{e}}_i\triangleq\boldsymbol{q}_i-\tilde{\boldsymbol{q}}_{0i}$ and $\hat{\boldsymbol{\Theta}}_i$ is the estimation of $\boldsymbol{\Theta}_i$.

To cope with the parameter uncertainty in (10), the update law of $\hat{\boldsymbol{\Theta}}_i$ is

$$\begin{aligned}\dot{\hat{\boldsymbol{\Theta}}}_i={}&-\boldsymbol{\Lambda}_i\boldsymbol{Y}_{i0}^{\mathrm{T}}(\boldsymbol{s}_{oi}-\tilde{\boldsymbol{s}}_i)+2\boldsymbol{\Lambda}_i^{1/2}\dot{\boldsymbol{Y}}_{i0}^{\mathrm{T}}\tilde{\boldsymbol{e}}_i-2\boldsymbol{\Lambda}_i\boldsymbol{Y}_{i0}^{\mathrm{T}}\times\\&\beta_1\left[\sum_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{q}_i-\boldsymbol{q}_j)+a_{i0}(\boldsymbol{q}_i-\boldsymbol{q}_0)\right],\\&i=1,\cdots,n.\end{aligned} \tag{17}$$

The highlight of the proposed control algorithms and the adaptive law is that the term $\tilde{\boldsymbol{s}}_i-\boldsymbol{s}_{oi}$ is elaborately designed to compensate the unavailability of the velocity measurements and $\tilde{\boldsymbol{e}}_i$ is introduced to deal with the followers' partial accessibility to the leader.

Theorem 1 If Assumptions 1 and 2 hold, the generalized graph contains a directed spanning tree rooted by the dynamic leader, and $\alpha_i>\|\boldsymbol{q}_0^{(i)}\|(i=1,2,3)$. Then using (4), (14), (16), (17) for (1), there exist parameters $\beta_1,\beta_2,\boldsymbol{\Lambda},\boldsymbol{K}_{1i},k_2$ and k_3 such that $\|\boldsymbol{q}_i(t)-\boldsymbol{q}_0(t)\|\to0$ and $\|\dot{\boldsymbol{q}}_i(t)-\dot{\boldsymbol{q}}_0(t)\|\to0, i=1,\cdots,n$, as $t\to\infty$.

Proof First, in order to solve the problem that only a subset of followers have access to the leader, the estimator (4) is used. From Lemma 2, it can be guaranteed that all the estimations of the leader's states converge to the real values in finite time. Therefore $\tilde{\boldsymbol{q}}_{0i}=\boldsymbol{q}_0$, $\tilde{\boldsymbol{v}}_{0i}=\dot{\boldsymbol{q}}_0$, and $\tilde{\boldsymbol{a}}_{0i}=\ddot{\boldsymbol{q}}_0$ when $t\geqslant T\triangleq\max\{T_1,T_2,T_3\}$. Hence

$$\tilde{\boldsymbol{s}}_i=\boldsymbol{s}_i=\dot{\boldsymbol{q}}_i-\dot{\boldsymbol{q}}_0+\beta_1\left[\sum_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{q}_i-\boldsymbol{q}_j)+a_{i0}(\boldsymbol{q}_i-\boldsymbol{q}_0)\right] \tag{18}$$

and (11) has the following form:

$$\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{s}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{s}+\Delta\boldsymbol{W}=\boldsymbol{\tau}-\boldsymbol{\tau}_0 \tag{19}$$

where $\boldsymbol{\tau}_0=\boldsymbol{Y}_d\boldsymbol{\Theta}=\boldsymbol{M}(\boldsymbol{q}_0)\ddot{\boldsymbol{q}}_0+\boldsymbol{C}(\boldsymbol{q}_0,\dot{\boldsymbol{q}}_0)\dot{\boldsymbol{q}}_0+\boldsymbol{G}(\boldsymbol{q}_0)$.

Then (15) can be written as

$$\begin{cases}\hat{\boldsymbol{q}}=\boldsymbol{z}-\beta_1(\boldsymbol{H}\otimes\boldsymbol{I}_p)\boldsymbol{e}+\beta_2(\boldsymbol{L}\otimes\boldsymbol{I}_p)\boldsymbol{e}_o+k_3\boldsymbol{e}_o\\\dot{\boldsymbol{z}}=\ddot{\boldsymbol{q}}_0-k_2(\boldsymbol{s}-\boldsymbol{s}_o)+k_3\beta_2(\boldsymbol{L}\otimes\boldsymbol{I}_p)\boldsymbol{e}_o\end{cases}. \tag{20}$$

Also, the term $\boldsymbol{s}-\boldsymbol{s}_o=\dot{\hat{\boldsymbol{q}}}_i-\dot{\boldsymbol{q}}_0+\beta_1\left[\sum_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{q}_i-\boldsymbol{q}_j)+a_{i0}(\boldsymbol{q}_i-\boldsymbol{q}_0)\right]-\beta_2\sum_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{e}_{oi}-\boldsymbol{e}_{oj})$ does not include any followers' velocity measurements. When $t>T$, consider the following Lyapunov function candidate

$$V(t)=\frac{1}{2}(-2\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Y}_d^{\mathrm{T}}\boldsymbol{e}+\boldsymbol{\Lambda}^{-\frac{1}{2}}\boldsymbol{\Theta}_e)^{\mathrm{T}}(-2\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Y}_d^{\mathrm{T}}\boldsymbol{e}+\boldsymbol{\Lambda}^{-\frac{1}{2}}\boldsymbol{\Theta}_e)+\frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{s}+\frac{1}{2}\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{s}_o+\frac{1}{2}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} \quad (21)$$

where $\boldsymbol{\Lambda}\triangleq\mathrm{diag}\{\boldsymbol{\Lambda}_1,\dots,\boldsymbol{\Lambda}_n\}$, and $\boldsymbol{\Theta}_e=\hat{\boldsymbol{\Theta}}-\boldsymbol{\Theta}$ is the estimation error of $\boldsymbol{\Theta}$. Taking the time derivative of $V(t)$ follows that

$$\dot{V}(t)=(-2\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Y}_d^{\mathrm{T}}\boldsymbol{e}+\boldsymbol{\Lambda}^{-\frac{1}{2}}\boldsymbol{\Theta}_e)^{\mathrm{T}}(-2\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Y}_d^{\mathrm{T}}\dot{\boldsymbol{e}}-2\boldsymbol{\Lambda}^{\frac{1}{2}}\dot{\boldsymbol{Y}}_d^{\mathrm{T}}\boldsymbol{e}+\boldsymbol{\Lambda}^{-\frac{1}{2}}\dot{\boldsymbol{\Theta}}_e)+\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{s}}+\frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\dot{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{s}+\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{s}}_o+\frac{1}{2}\boldsymbol{s}_o^{\mathrm{T}}\dot{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{s}_o+\boldsymbol{e}^{\mathrm{T}}\dot{\boldsymbol{e}}. \quad (22)$$

Combining (16) and (19), when $t\geqslant T$, notice that $\tilde{\boldsymbol{q}}_{0i}=\boldsymbol{q}_0$, $\tilde{\boldsymbol{v}}_{0i}=\dot{\boldsymbol{q}}_0$, and $\tilde{\boldsymbol{a}}_{0i}=\ddot{\boldsymbol{q}}_0$, thus the error dynamics of the system can be written as

$$\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{s}}=-\boldsymbol{K}_1(\boldsymbol{s}-\boldsymbol{s}_o)-\Delta\boldsymbol{W}+\boldsymbol{Y}_d\boldsymbol{\Theta}_e-\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{s}-2\boldsymbol{\Lambda}\boldsymbol{Y}_d\boldsymbol{Y}_d^{\mathrm{T}}\boldsymbol{e}. \quad (23)$$

Differentiating the first equation of the observer (20), and substituting the second one into it, we have

$$\dot{\boldsymbol{s}}_o=\dot{\boldsymbol{s}}-k_2(\boldsymbol{s}_o-\boldsymbol{s})-k_3\boldsymbol{s}_o. \quad (24)$$

Note the fact that $\dot{\boldsymbol{\Theta}}_e=\dot{\hat{\boldsymbol{\Theta}}}$. Then, substituting (17), (23) and (24) into (22), from Property 2, we can get

$$\dot{V}(t)=(-2\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Y}_d^{\mathrm{T}}\boldsymbol{e}+\boldsymbol{\Lambda}^{-\frac{1}{2}}\boldsymbol{\Theta}_e)^{\mathrm{T}}[-2\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Y}_d^{\mathrm{T}}\dot{\boldsymbol{e}}-\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Y}_d^{\mathrm{T}}(\boldsymbol{s}_o-\boldsymbol{s})-2\boldsymbol{\Lambda}^{\frac{1}{2}}\beta_1\boldsymbol{Y}_d^{\mathrm{T}}(\boldsymbol{H}\otimes\boldsymbol{I}_p)\boldsymbol{e}]-\boldsymbol{K}_1\boldsymbol{s}^{\mathrm{T}}(\boldsymbol{s}-\boldsymbol{s}_o)-\boldsymbol{s}^{\mathrm{T}}\Delta\boldsymbol{W}-\boldsymbol{s}^{\mathrm{T}}\boldsymbol{Y}_d\boldsymbol{\Theta}_e-2\boldsymbol{\Lambda}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{Y}_d\boldsymbol{Y}_d^{\mathrm{T}}\boldsymbol{e}+\frac{1}{2}\boldsymbol{s}_o^{\mathrm{T}}\dot{\boldsymbol{M}}\boldsymbol{s}_o-\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{C}\boldsymbol{s}_o+\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{C}\boldsymbol{s}_o+\boldsymbol{s}_o^{\mathrm{T}}[-\boldsymbol{K}_1(\boldsymbol{s}-\boldsymbol{s}_o)-\Delta\boldsymbol{W}+\boldsymbol{Y}_d\boldsymbol{\Theta}_e-\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{s}-2\boldsymbol{\Lambda}\boldsymbol{Y}_d\boldsymbol{Y}_d^{\mathrm{T}}\boldsymbol{e}]-k_3\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}\boldsymbol{s}_o-k_2\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{s}_o-\boldsymbol{s})+\boldsymbol{e}^{\mathrm{T}}\dot{\boldsymbol{e}}. \quad (25)$$

After performing some basic algebraic operations and considering Lemma 3, (25) can be simplified as

$$\dot{V}(t)\leqslant\beta_1\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{H}\boldsymbol{s}-\beta_1^2\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{e}+b_{11}\|\boldsymbol{s}\|^2+b_{12}\|\boldsymbol{s}\|\|\boldsymbol{e}\|+b_{13}(\|\boldsymbol{s}\|^2\|\boldsymbol{e}\|+\beta_1\overline{\sigma}(\boldsymbol{H})\|\boldsymbol{s}\|\|\boldsymbol{e}\|^2)+\beta_2\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}\boldsymbol{L}\boldsymbol{s}-\beta_2^2\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{e}+b_{21}\|\boldsymbol{s}\|\|\boldsymbol{s}_o\|+b_{22}\|\boldsymbol{s}_o\|\|\boldsymbol{e}\|+b_{23}(\|\boldsymbol{s}\|\|\boldsymbol{s}_o\|\|\boldsymbol{e}\|+\beta_2\overline{\sigma}(\boldsymbol{L})\times\|\boldsymbol{s}_o\|\|\boldsymbol{e}\|^2)-\boldsymbol{K}_1\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}-\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{C}\boldsymbol{s}+\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{C}\boldsymbol{s}_o+\boldsymbol{K}_1\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{s}_o-k_2\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}\boldsymbol{s}_o+k_2\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}\boldsymbol{s}-k_3\boldsymbol{s}_o^{\mathrm{T}}\boldsymbol{M}\boldsymbol{s}_o+\boldsymbol{e}^{\mathrm{T}}\boldsymbol{s}-\beta_1\boldsymbol{e}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{e}. \quad (26)$$

To facilitate the subsequent analysis, the following equalities are introduced to deal with the cross terms such as $\|\boldsymbol{s}\|\|\boldsymbol{e}\|^2$ and $\|\boldsymbol{s}_o\|\|\boldsymbol{e}\|^2$.

$$\|\boldsymbol{s}\|\|\boldsymbol{e}\|^2=-\frac{1}{2}\|\boldsymbol{e}\|^2(1-\|\boldsymbol{s}\|)^2+\frac{1}{2}\|\boldsymbol{e}\|^2+\frac{1}{2}\|\boldsymbol{e}\|^2\|\boldsymbol{s}\|^2$$
$$\|\boldsymbol{s}_o\|\|\boldsymbol{e}\|^2=-\frac{1}{2}\|\boldsymbol{e}\|^2(1-\|\boldsymbol{s}_o\|)^2+\frac{1}{2}\|\boldsymbol{e}\|^2+\frac{1}{2}\|\boldsymbol{e}\|^2\|\boldsymbol{s}_o\|^2 \quad (27)$$

Then, it can be deduced that

$$\dot{V}(t)\leqslant(p_1+b_{13}\|\boldsymbol{e}\|+\frac{1}{2}b_{23}\|\boldsymbol{e}\|)\|\boldsymbol{s}\|^2+(p_2+\frac{1}{2}b_{23}\|\boldsymbol{e}\|+\frac{1}{2}b_{23}\beta_2\overline{\sigma}(\boldsymbol{L})\|\boldsymbol{e}\|^2)\|\boldsymbol{s}_o\|^2+(p_4-\frac{1}{2}b_{23}\|\boldsymbol{e}\|)(\|\boldsymbol{s}_o\|-\|\boldsymbol{s}\|)^2+D \quad (28)$$

where

$$p\triangleq\begin{cases}p_1=-\underline{\lambda}_{\boldsymbol{K}_1}+\frac{1}{2}k_{\overline{m}}k_3+\omega_1\\ p_2=-k_{\underline{m}}(k_2+k_3)+\overline{\lambda}_{\boldsymbol{K}_1}+\omega_2\\ p_3=-\beta_1\left[\underline{\sigma}(\boldsymbol{H})-\frac{1}{2}b_{13}\overline{\sigma}(\boldsymbol{H})\right]+\omega_3\\ p_4=-\frac{1}{2}\beta_2k_{\overline{m}}\overline{\sigma}(\boldsymbol{L})-\frac{1}{2}b_{21}-\frac{1}{2}k_{\overline{m}}k_3\end{cases}$$

$$D=\Big[-\frac{1}{2}\beta_2^2k_{\overline{m}}\overline{\lambda}_{\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}}-\frac{1}{2}\beta_1^2k_{\overline{m}}\overline{\lambda}_{\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}}-\frac{1}{2}-\frac{1}{2}b_{12}\Big]\times(\|\boldsymbol{s}\|-\|\boldsymbol{e}\|)^2-\frac{1}{2}\beta_1b_{13}\overline{\sigma}(\boldsymbol{H})\|\boldsymbol{e}\|^2(1-\|\boldsymbol{s}\|)^2-\frac{1}{2}b_{22}(\|\boldsymbol{s}_o\|-\|\boldsymbol{e}\|)^2-\frac{1}{2}\beta_2b_{23}\overline{\sigma}(\boldsymbol{L})\times\|\boldsymbol{e}\|^2(1-\|\boldsymbol{s}_o\|)^2$$

$\underline{\sigma}(\boldsymbol{X})$ denotes the minimum singular value of the matrix $\boldsymbol{X}$. $\overline{\lambda}_{\boldsymbol{X}}$ and $\underline{\lambda}_{\boldsymbol{X}}$ denote the largest and the smallest eigenvalues of the matrix $\boldsymbol{X}$, respectively. From Assumptions 1 and 2 and Property 1, we can get that $\overline{\sigma}(\boldsymbol{C})$ is bounded. And w_1,w_2,w_3 in p are as follows:

$$w_1=\beta_1k_{\overline{m}}\overline{\sigma}(\boldsymbol{H})+b_{11}+\frac{1}{2}b_{12}+\frac{1}{2}\beta_1b_{13}\overline{\sigma}(\boldsymbol{H})+\frac{1}{2}b_{21}+\frac{1}{2}\beta_2k_{\overline{m}}\overline{\sigma}(\boldsymbol{C})+\frac{1}{2}\beta_1^2k_{\overline{m}}\overline{\lambda}_{\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}}+\frac{1}{2}$$
$$w_2=\frac{1}{2}b_{21}+\frac{1}{2}b_{22}+\frac{1}{2}b_{23}\overline{\sigma}(\boldsymbol{L})+\frac{3}{2}\overline{\sigma}(\boldsymbol{C})+$$

$$\frac{1}{2}\beta_2 k_{\overline{m}}\overline{\sigma}(\boldsymbol{L})+\frac{1}{2}\beta_2^2 k_{\overline{m}}\overline{\lambda}_{\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}}$$

$$w_3=\frac{1}{2}\beta_1^2 k_{\overline{m}}\overline{\lambda}_{\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}}+\frac{1}{2}\beta_2^2 k_{\overline{m}}\overline{\lambda}_{\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}}+$$

$$\frac{1}{2}\beta_2 b_{23}\overline{\sigma}(\boldsymbol{L})+\frac{1}{2}b_{21}+\frac{1}{2}b_{22}+\frac{1}{2}.$$

It is clear that $D\leqslant 0$, thus $\dot{V}(t)\leqslant 0$ if the following inequalities hold:

$$\begin{cases} p_1+b_{13}\|\boldsymbol{e}\|+\frac{1}{2}b_{23}\|\boldsymbol{e}\|\leqslant 0 & \text{(29a)}\\ p_2+\frac{1}{2}b_{23}\|\boldsymbol{e}\|+\frac{1}{2}b_{23}\beta_2\overline{\sigma}(\boldsymbol{L})\|\boldsymbol{e}\|^2\leqslant 0 & \text{(29b)}\\ p_3\leqslant 0 & \text{(29c)}\\ p_4-\frac{1}{2}b_{23}\|\boldsymbol{e}\|\leqslant 0 & \text{(29d)}\end{cases}$$

Obviously, $p_4-\frac{1}{2}b_{23}\|\boldsymbol{e}\|\leqslant 0$ for $\|\boldsymbol{e}\|\geqslant 0$. Note that (29a) and (29b) contain the term $\|\boldsymbol{e}\|$ and $\|\boldsymbol{e}\|^2$, they have solutions only when

$$p_1\leqslant 0,\quad p_2\leqslant 0. \tag{30}$$

Combining (29) and (30) makes $\dot{V}(t)$ be negative semi-definite when the parameters satisfy the following inequalities:

$$\begin{cases} \underline{\lambda}_{\boldsymbol{K}_1}\geqslant\frac{1}{2}k_{\overline{m}}k_3+\omega_1 & \text{(31a)}\\ k_2+k_3\geqslant\frac{\overline{\lambda}_{\boldsymbol{K}_1}}{k_{\underline{m}}}+\frac{\omega_2}{k_{\underline{m}}} & \text{(31b)}\\ \beta_1\geqslant\frac{\omega_3}{\underline{\sigma}(\boldsymbol{H})-\frac{1}{2}b_{13}\overline{\sigma}(\boldsymbol{H})} & \text{(31c)}\end{cases}$$

$\dot{V}(t)\leqslant 0$ implies that terms $\boldsymbol{s},\boldsymbol{s}_o$, and $\boldsymbol{e}$ together with their derivatives are bounded. By differentiating (25), we can see that $\ddot{V}(t)$ is bounded. Thus $\dot{V}(t)$ is uniformly continuous. It can be concluded from Barbalat's Lemma [27] that $\dot{V}(t)\to 0$, as $t\to\infty$, i.e., $\boldsymbol{s}(t)\to 0$, $\boldsymbol{s}_o(t)\to 0$, and $\boldsymbol{e}(t)\to 0$, as $t\to\infty$. Therefore, $\|\boldsymbol{q}_i(t)-\boldsymbol{q}_0(t)\|\to 0$, and $\|\dot{\boldsymbol{q}}_i(t)-\dot{\boldsymbol{q}}_0(t)\|\to 0, i=1,\dots,n$, as $t\to\infty$ □

Remark 2 Equation (9) can be written as $\dot{\boldsymbol{e}}_o=-\beta_2(\boldsymbol{L}\otimes\boldsymbol{I}_p)\boldsymbol{e}_o+\boldsymbol{s}_o$. Since the interaction of the agents forms a directed spanning graph, we know from the Lasalle invariance principle and the theory of input-to-state stable (ISS) [27] that the state $\boldsymbol{e}_o$ would converge to zero if $\boldsymbol{s}_o$ converges to zero. Therefore, we also have $\|\dot{\boldsymbol{q}}_i(t)-\hat{\dot{\boldsymbol{q}}}_i(t)\|\to 0$, as $t\to\infty$, which means that we can observe the velocities only using position measurements.

Remark 3 It is worthy to mention that the term $\tilde{\boldsymbol{s}}_i-\boldsymbol{s}_{oi}$ plays a critical role in the observer (14), the control algorithm (16), and the adaptive law (17). The elaborately designed term $\tilde{s}_i-\boldsymbol{s}_{oi}$ frees the use of velocity sensors. Meanwhile, this term also introduces other sources of velocity information, such as the follower itself and the estimations of the leader, which facilitate the design of the control law and the following stability analysis.

4. Simulation results

Numerical simulations are presented in this section to demonstrate the effectiveness of the proposed control algorithm. Consider four networked two-link manipulators modeled by Euler-Lagrange equations, in which

$$\boldsymbol{M}_i(\boldsymbol{q}_i)=$$

$$\begin{bmatrix}\boldsymbol{\theta}_{i(1)}+2\boldsymbol{\theta}_{i(2)}\cos(\boldsymbol{q}_{i(2)}) & \boldsymbol{\theta}_{i(3)}+\boldsymbol{\theta}_{i(2)}\cos(\boldsymbol{q}_{i(2)})\\ \boldsymbol{\theta}_{i(3)}+\boldsymbol{\theta}_{i(2)}\cos(\boldsymbol{q}_{i(2)}) & \boldsymbol{\theta}_{i(3)}\end{bmatrix}$$

$$\boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i)=$$

$$\begin{bmatrix}-\boldsymbol{\theta}_{i(2)}\sin(\boldsymbol{q}_{i(2)})\dot{\boldsymbol{q}}_{i(2)} & -\boldsymbol{\theta}_{i(2)}\sin(\boldsymbol{q}_{i(2)})(\dot{\boldsymbol{q}}_{i(1)}+\dot{\boldsymbol{q}}_{i(2)})\\ \boldsymbol{\theta}_{i(2)}\sin(\boldsymbol{q}_{i(2)})\dot{\boldsymbol{q}}_{i(1)} & 0\end{bmatrix}$$

$$\boldsymbol{G}_i(\boldsymbol{q}_i)=\begin{bmatrix}\boldsymbol{\theta}_{i(4)}g\cos(\boldsymbol{q}_{i(1)})+\boldsymbol{\theta}_{i(5)}g\cos(\boldsymbol{q}_{i(1)}+\boldsymbol{q}_{i(2)})\\ \boldsymbol{\theta}_{i(5)}g\cos(\boldsymbol{q}_{i(1)}+\boldsymbol{q}_{i(2)})\end{bmatrix}$$

where $\boldsymbol{q}_i\triangleq[\boldsymbol{q}_{i(1)},\boldsymbol{q}_{i(2)}]^{\mathrm{T}}, g=9.8\ \mathrm{m/s^2}$ is the acceleration of gravity, $\boldsymbol{\theta}_i\triangleq[\boldsymbol{\theta}_{i(1)},\boldsymbol{\theta}_{i(2)},\boldsymbol{\theta}_{i(3)},\boldsymbol{\theta}_{i(4)},\boldsymbol{\theta}_{i(5)}]=[m_1l_{c1}^2+m_2(l_1^2+l_{c2}^2)+J_1+J_2, m_2l_1l_{c2}, m_2l_{c2}^2+J_2, m_1l_{c1}+m_2l_1, m_2l_{c2}]$. For simplicity, we choose identical joint arms for the four followers. Let the masses of link 1 and link 2 be, respectively, $m_1=0.5$ kg, and $m_2=0.4$ kg, the lengths of link 1 and link 2 be, respectively, $l_1=0.4$ m, and $l_2=0.3$ m, the distances of the mass center of link 1 and link 2 between neighbors be, respectively, $l_{c1}=0.2$ m, and $l_{c2}=0.15$ m. In addition, the moments of inertia of link 1 and link 2 are, respectively, $J_1=0.006\,7\ \mathrm{kg\cdot m^2}$, and $J_2=0.003\ \mathrm{kg\cdot m^2}$.

The interaction topology between the followers and the leader are shown in Fig. 1, in which only agent 1 can directly receive information form the leader. Here, an arrow from i to j denotes the agent j can receive information form the agent i $(i,j=0,1,\dots,4)$. The rod-shaped topology is the most sparse directed graph, which implies that the minimum amount of information transmits in this topology than that in other topologies with the same number of agents. The initial positions of the followers are chosen as $\boldsymbol{q}_i(0)=[(\pi/7)i,(\pi/8)i]^{\mathrm{T}}$, and the velocity observations of the four followers $\hat{\dot{\boldsymbol{q}}}(0)$ are set as $[0,0]^{\mathrm{T}}$. Let the reference states of the leader be $\boldsymbol{q}_0(t)=[\sin(t),-\sin(t)]^{\mathrm{T}}$, and hence the angular velocity be $\dot{\boldsymbol{q}}_0(t)=[\cos(t),-\cos(t)]^{\mathrm{T}}$. The control parameters are set as $\alpha_1=\alpha_2=\alpha_3=1.5$, $\beta_1=5$, $\beta_2=1$, $\boldsymbol{\Lambda}_i=0.2\boldsymbol{I}_2$ and $\boldsymbol{K}_{1i}=12\boldsymbol{I}_2, k_2=1, k_3=12$. Fig. 2–Fig. 4 illustrate the evolution of the distributed tracking process. Fig. 2 shows the initial states of the agents. In Fig. 3, it can be seen that agent 1 has almost caught up with the leader and others approach the leader gradually. Then, after about 2 s, all the followers can trace and follow the motion of the leader, and then maintain the performance hereafter, which is shown in Fig. 4, namely, they have realized target tracking.

Fig. 1 Network topology associated with leader 0 and four followers

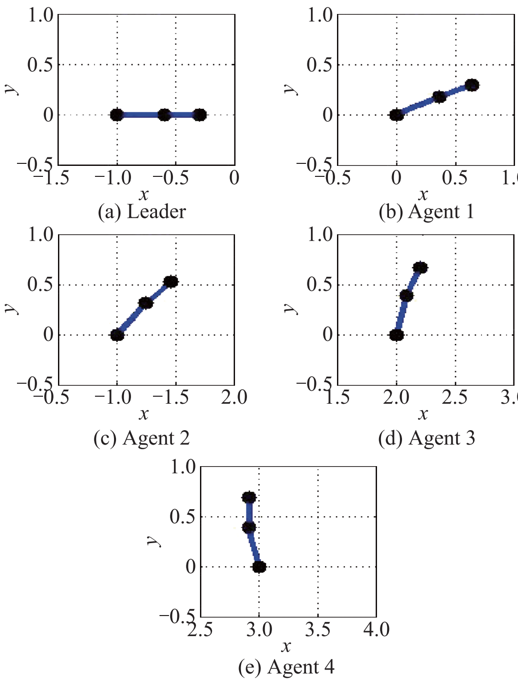

Fig. 2 Motion of robotic arms at $t = 0$ s

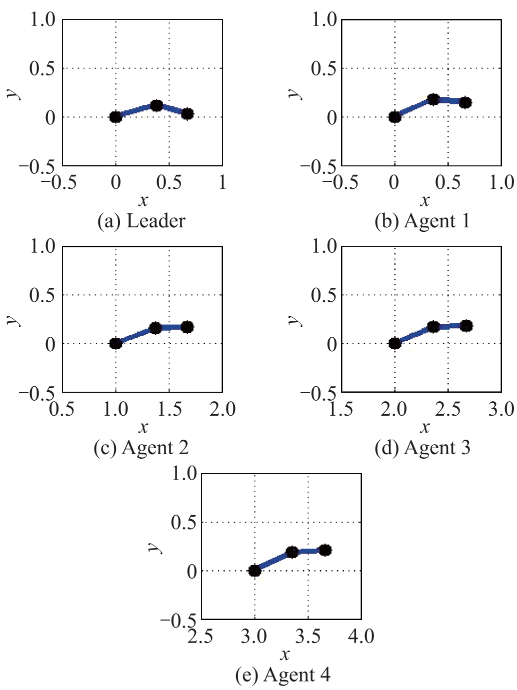

Fig. 3 Motion of robotic arms at $t = 0.3$ s

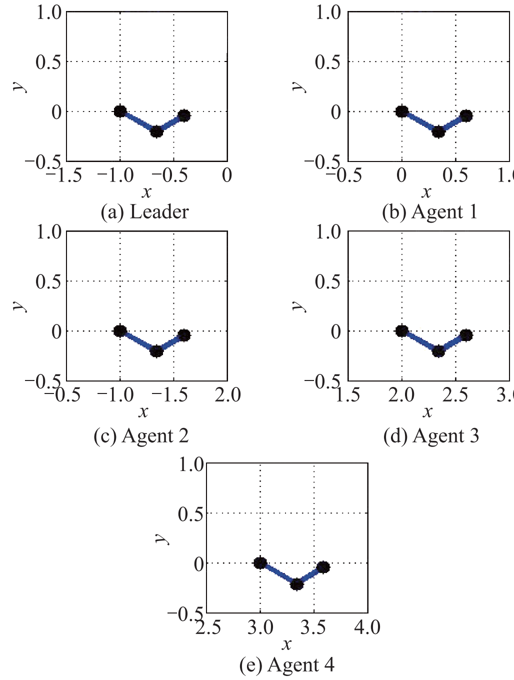

Fig. 4 Motion of robotic arms at $t = 10$ s

Fig. 5–Fig. 6 show the variations of angle and angular velocity during the tracking process. These figures indicate that good tracking performance is achieved. Fig. 7–Fig. 9 show the corresponding state errors during the whole tracking process. From Fig. 7, it is found that the angle differences between the followers and the leader converge to zero after slight chattering of 2 s.

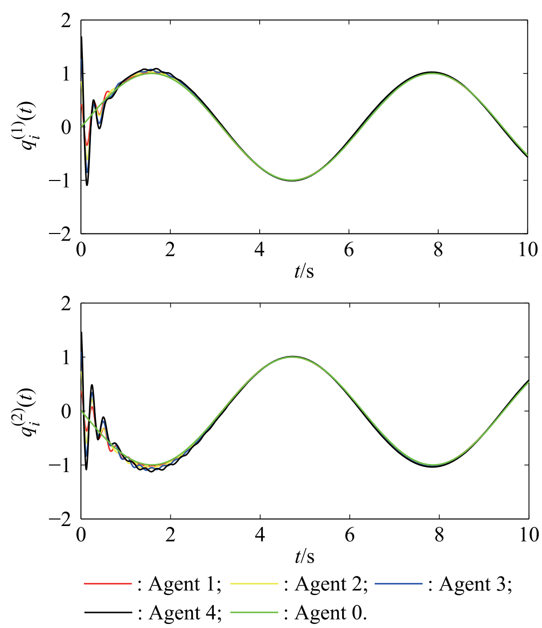

Fig. 5 Trajectories of angle $q_i(t)$

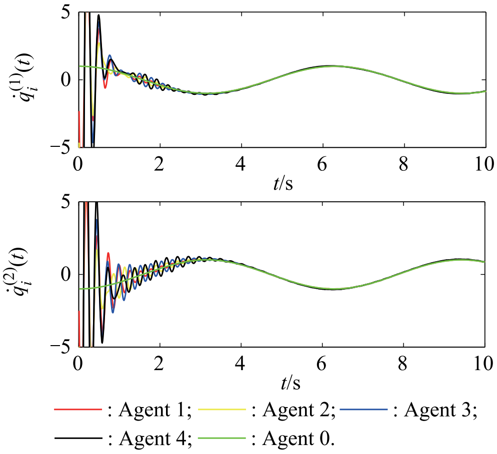

Fig. 6 Trajectories of angular velocity $\dot{q}_i(t)$

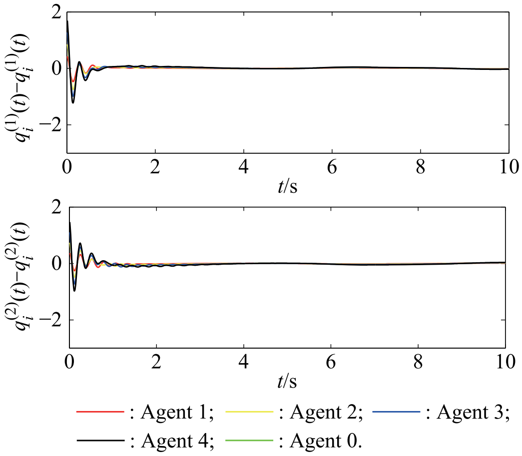

Fig. 7 Angle tracking errors between the followers and the leader

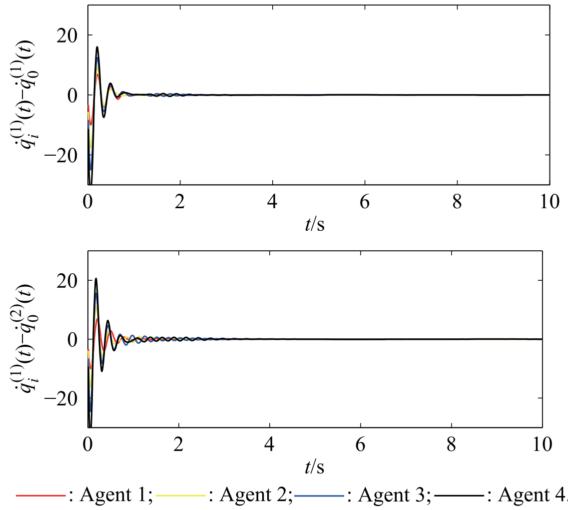

Fig. 8 Angular velocity tracking errors between the followers and the leader

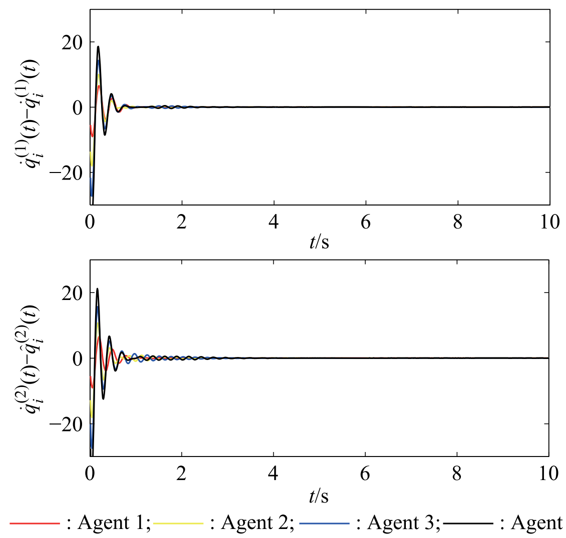

Fig. 9 Angular velocity errors between the observations and the real values

Accordingly, Fig. 8 illustrate the angular velocity differences between the followers and the leader. Then, we can easily draw the conclusion that the distributed tracking to a dynamic leader is achieved effectively by using the proposed control laws. Besides, the errors between the real values of the angular velocity and their observations obtained by the followers are shown in Fig. 9, which imply that the proposed distributed velocity observers yield accurate observations, and it makes important sense to compensate the absence of velocity measurements. All the simulation results demonstrate the effectiveness and feasibility of the proposed framework of the distributed observer-controller.

In order to demonstrate the effectiveness of our proposed method, some comparative simulations are also carried out under the conditions in [13] which are more rigid than ours. At the beginning of the tracking process, vibration occurs due to the output of the signum functions and the regulation of the observers. However after the distributed sliding mode estimators yielding precise estimations of the leader's states, the vibration vanishes and all the followers can track the leader as accurately as that in [13]. Due to the space limitations, the figures which demonstrate the comparative results are not presented here.

5. Conclusion

The problem of distributed coordinated tracking for multiple networked Euler-Lagrange systems with parameter uncertainties has been investigated under the constraint that the leader is a neighbor of only parts of the followers on a directed graph containing a spanning tree. Well-designed distributed velocity observers fully utilize all the available local information and the estimated reference information to cope with the unavailability of velocity information. Consequently, a new framework of observer-controller combined with the adaptive law is proposed to solve the coordinated tracking problem by using only position information. Several sufficient conditions which guarantee the stability of the system are provided and the effectiveness of the proposed control strategies is verified by simulation results. It should be pointed out that we just solve the tracking problem with no-time delay in the communication. Future works include the study of the cooperative tracking of multiple Euler-Lagrange systems with time delays under a switching directed topology.

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