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On Consensus of Multiple High-order Uncertain Systems Based on Distributed Backstepping Framework

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Abstract This paper focuses on the consensus problem of multiple high-order systems with uncertainties. Since it is difficult to use matrix theory approaches to design consensus controllers for a class of multiple high-order uncertain nonlinear systems, in this paper a set of consensus control laws are proposed by employing adaptive control theory and a backstepping technique. The distributed virtual control functions of the multi-agent systems are elaborately constructed by only using their local information in the recursive controller design procedure. Furthermore, the asymptotic stability of the overall interconnected system is proved relying on the Lyapunov stability analysis method. Finally, simulations are provided to verify the effectiveness of the control algorithms.

Keywords Consensus, Backstepping, High-order Nonlinear Systems, Multi-agent, Distributed Control

1. Introduction

Over the past decade in particular, the cooperative control of multi-agent systems has received increasing attention given the fact that many benefits can be obtained when a single complicated agent is equivalently replaced by multiple simpler agents. Numerous results have been obtained to solve a variety of multi-agent cooperative control problems [1-6,13-17]. In the distributed control of a group of autonomous agents, the main objective typically is to have the whole group of agents working in a cooperative fashion throughout a distributed protocol. These controllers are distributed in the sense that the controller design for each agent only requires relative state information between itself and its neighbours. Hence, coordination refers to a close relationship among all agents in the group where information sharing plays a crucial role. The distributed control scheme has many advantages in achieving cooperative group performances, especially with low operational costs, fewer system requirements, higher robustness, stronger adaptivity, and flexible scalability. The control theory of multi-agent systems can be applied in many practical engineering applications, such as the cooperative control of unmanned ground/air/underwater vehicles [7-9], distributed sensor networks [10], aggregation and rendezvous control [11], the attitude alignment of spacecraft [12], and so on. Therefore, the cooperative control of multi-agent systems has been widely recognized and will be universally appreciated in the future.

Among the existing works mentioned above, most of them have studied only first- and second-order dynamics. Recently, some researchers turned their focus on higher-order multi-agent systems coordination. One motivation for studying higher-order consensus comes from observing the behaviour of flocks of birds [18]. It is often noted that such flocks fly in formation, maintaining a nominal separation from each other, but each travelling with the same velocity vector. However, sometimes a bird flock will abruptly change direction, perhaps when one of them suddenly perceives a source of danger or food. Clearly, the birds in this setting need to build consensus as to not only their relative positions and velocities, but also to their acceleration. In [19], Dong considered a group of third-order nonlinear systems with parameter uncertainties using a backstepping technique and an adaptive control method. However, the proposed control law does not work when the order of systems is larger than three. It is worth noting that the control algorithm in [19] cannot easily be extended to high-order nonlinear systems with uncertainties at each step. In [19], since the adaptation law (17) contains its neighbours' information, the control law (18) has two-hop information. When the order of the system is n, $n-1$ hop information is needed to design control laws. This is unavailable, obviously. In addition, in many practical engineering applications, many systems are modelled by higher-order dynamics, for example, the jerk systems, described by third-order differential equations [20]. A single-link flexible joint manipulator is well modelled by a fourth-order nonlinear system [21-23]. It is not acceptable to model the plant dynamics with only single- or double-integrator dynamics. Hence, it is particularly important to extend the coordination problem from lower-order dynamics to higher-order ones. Compared with the first-order and second-order dynamics, the higher-order ones involve more details relating to the interactions between the system dynamics (states and their derivatives) and the communication network. Up to now, most of the existing literature has only been concerned the high-order integrators with a linear strict-feedback form ([18,24-26],etc.), which are mainly based on the matrix theory on graphs. In [18], Ren et al. showed a matrix approach-based framework for high-order integrators multi-agent systems. They defined a class of l-order ($l >= 3$) consensus algorithms and showed necessary and sufficient conditions under which each information variable and their higher-order derivatives converged to common values. Jiang [24] investigated the consensus problem for multi-agent systems with individual agents modelled by high-order integrators under a fixed/switching topology and zero/non-zero communication time-delays. In [25], the consensus of high-order integrators multi-agent systems with time-delays and switching topologies was studied. The coordination of high-order linear systems with disturbances was investigated in [26,27]. Discrete-time high-order linear multi-agent systems control problem was considered in [28], and the results for the general high-order linear time-invariant (LTI) systems were published in [29,30]. Huang et al. summarized a survey of recent progress in the study of distributed high-order linear multi-agent coordination in [15].

As for the consensus of multiple high-order nonlinear systems, only a few results have been proposed. Dong et al.[31] considered the tracking control problem. Distributed robust/adaptive control laws were proposed such that the states of each system converged to the desired trajectory asymptotically. However, the model of the systems is without uncertainties. Because of the inherent characteristics of multiple linear systems, matrix theory approaches are frequently used in stability analysis. However, in many practical applications, the dynamics of the systems are not only nonlinear but also have uncertainties, thus solving consensus problems for multiple high-order uncertain nonlinear systems, which would make great sense for practical applications. Matrix theory-based frameworks are not applicable in many scenarios, especially for nonlinear systems. Thus, the consensus control of high-order nonlinear systems with uncertainties is more challenging than that of certain high-order linear ones. The extension of adaptive control to high-order dynamics is not straightforward because of the growth in the order. The challenge is to make sure that both the control protocols and the parameter update laws are distributed - that is, they are allowed to depend only on locally available information about the agent and its neighbours. High-order systems contain more states and their derivatives, so the design of adaptive control becomes more complicated. This requires the careful crafting of a suitable Lyapunov function which automatically yields a distributed adaptive controller that depends only on local information. In [32], Dong first considered the adaptive consensus seeking of a class of multiple nonlinear systems using backstepping techniques. He considered two consensus problems, one with a constant agreement value and another with a reference system whose state is only available to a portion of the agents.

In this paper, we consider a class of high-order nonlinear multi-agent systems with uncertain parameters in the nth-order terms. We tried to solve the control problem under a distributed backstepping framework and we propose a detailed design process. The key to designing distributed controllers is the selection of a sequence of suitable Lyapunov functions and the adaptive laws that depend upon the communication network and the dynamics of the system. The basis for the selection of suitable graph-dependent Lyapunov functions was set out in the backstepping technique on the graph. A distributed recursive design approach is proposed to achieve the consensus of multiple high-order nonlinear systems with uncertainties. The main tasks of this paper include: 1) This paper reviews the major results and progress in distributed high-order multi-agent coordination. A type of multiple high-order nonlinear system with uncertainties is considered. 2) Distinct from the conventional matrix theory-based frameworks, a systematic controller design method/framework is proposed by combining a distributed backstepping method with adaptive control techniques. Furthermore, the convergence of the system errors is proven rigorously by virtue of the Lyapunov stability theory and Barbalat's lemma. 3) In the existing literature, only chained systems whose order is lower than three were used as the representative high-order systems, while a multiple four-order nonlinear uncertain system is implemented in this paper. A group of 11 agents is used to verify the validity of the distributed controller.

The subsequent sections are organized as follows: In section 2, the consensus problem is formally stated and the background as well as the necessary preliminaries concerning the control problem are given. In section 3, the cooperative control laws are proposed relying upon the backstepping method. The uncertainties of the parameters are addressed by distributed adaptive control laws. In section 4, simulations of the consensus control for multiple four-order uncertain nonlinear systems are provided to demonstrate the performance of the proposed control laws. The last section concludes the paper.

2. Preliminaries and Problem Statement

In this section, basic graph theory for multi-agent system control and the control problem are introduced.

2.1. Basic Graph Theory for a Multi-agent System

A team of m high-order nonlinear systems labelled as systems 1 to m are considered. The communication topology among the m systems is assumed to be bi-directional and the interactions among the nodes are represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V}$ is a set of the indices of the systems and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges that describe the communications between the agents. If $(i, j) \in \mathcal{E}$, then i is a neighbour of j, meaning that system j can obtain information from system i. $\mathcal{A}$ is a weighted adjacency matrix with non-negative adjacency elements a_{ij}. Moreover, it is assumed that $a_{ii} = 0$. If the state of system i is available to system j, then system i is said to be a neighbour of system j. The neighbour set of node v_j is denoted by $\mathcal{N}_j$, where $j \notin \mathcal{N}_j$.

Assumption 1. The communication graph $\mathcal{G}$ is fixed and connected.

For the communication graph $\mathcal{G}$ with the weighted adjacency matrix $\mathcal{A} = [a_{ij}]_{m\times m}$, satisfying $a_{ij} = a_{ji} > 0$. Its Laplacian matrix $L = [L_{ij}]$ is defined as:

$$L_{ij} = \begin{cases} -a_{ij} & \text{if } i \neq j \text{ and } i \in \mathcal{N}_j \\ \sum\limits_{l\in\mathcal{N}_j} a_{jl} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

2.2. Problem Statement

In this paper, we consider multi-agent systems formed by a class of m high-order uncertain nonlinear systems in chained form. The topology of the information exchange among the systems is described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. The novel dynamics of the j-th system is expressed as follows:

$$\dot{x}_{ij} = x_{(i+1)j} \tag{2}$$
$$\dot{x}_{nj} = u_j + \phi_j^T(x_{1j}, x_{2j}, \ldots, x_{nj})\theta_j \tag{3}$$

where $i = 1, 2, \ldots, n-1$ denotes the orders of each system, and $j = 1, 2, \ldots, m$ denotes the index number of the systems, and $x_j = [x_{1j}, x_{2j}, \ldots, x_{nj}]^T \in \mathbf{R}^n$ denotes the state of the j-th system. $u_j \in \mathbf{R}$ is the control input of the j-th system. $\theta_j \in \mathbf{R}^p$ denotes the uncertain parameter vector, where p is a positive integer. The function $\phi_j(x_{1j}, x_{2j}, \ldots, x_{nj}) \in \mathbf{R}^p$ is smooth and assumed to be known.

The aim of this paper is to design a control law for the j-th system based on its own local states information when the communication topology is fixed and connected, such that:

$$|x_{1j} - x_{1l}| \to 0, \text{ as } t \to \infty \text{ for } j, l = 1, \ldots, m. \tag{4a}$$
$$x_{ij} \to 0, \text{ as } t \to \infty \text{ for } i = 2, \ldots, n. \tag{4b}$$

3. Distributed Control Law Design

In this section, basic definitions are given and the local neighbourhood virtual controllers are introduced. Some requirements on the topology are laid out, and a series of Lyapunov functions are given. Finally, a detailed design procedure is given based on the recursive framework.

3.1. Local Neighbourhood Virtual Controllers

The high-order dynamics involve more details related to the interactions between the system dynamics and the communication network, which are reflected in the virtual controls and the Lyapunov functions. The m systems in (2)-(3) include strict-feedback forms. Owing to the structural character of the lower-triangular strict-feedback system, the high-rank state of each differential equation is used as virtual control. In this way, the consensus control problem for the higher order multiple systems can be broken into a sequence of design problems for multiple lower order subsystems. The extension of adaptive backstepping control to distributed multiple higher-order dynamics is not straightforward. In a cooperative adaptive controller for multi-agent systems, it is required that not only the backstepping control laws but also the adaptive laws should be distributed and designed relying on their local information.

Definition 1. *We define a set of new variables* $z_{*j} = [z_{1j}, z_{2j}, \ldots, z_{nj}]^T$ *with the aid of the backstepping technique as follows:*

$$z_{1j} = x_{1j} \tag{5}$$
$$z_{ij} = x_{ij} - \alpha_{ij}, 2 \leq i \leq n \tag{6}$$

where $j = 1, \ldots, m$. α_{ij} *is the virtual control function which is to be elaborately designed through the recursive backstepping method.*

3.2. Recursive Controller Design Procedure

Distinct from centralized methods, the decentralized recursive backstepping method using only the local information is designed based on the sequence of virtual controllers which are also necessarily designed relying on local information. The aim of this work is to design the virtual controller not only in a recursive way, like other ordinary backstepping methods, but also in a distributed manner, which makes the design procedure much more difficult than the centralized methods. Furthermore, the previously proposed design scheme cannot be extended to deal with this case due to the complex intrinsic nonlinearity defined in (3). The actual controllers u_j can be derived from α_{nj} after α_{nj} is designed. The detailed recursive design procedure is given as follows.

In the first step, α_{2j} is used to denote the first-order virtual controller of system j. Using (2) for (5), we can derive that:

$$\dot{z}_{1j} = z_{2j} + \alpha_{2j} \tag{7}$$

Consider the first-order error variable $z_{1j} = x_{1j}$ of the first-order subsystem of (2)-(3), and choose the Lyapunov function candidate V_1 as follows:

$$V_1 = \frac{1}{2} z_{1*}^T z_{1*} \tag{8}$$

where $z_{1*} = [z_{11}, z_{12}, \ldots, z_{1m}]^T$.

Taking the time-derivative of V_1 and following (6) and (7), we can obtain:

$$\dot{V}_1 = \sum_{j=1}^{m} z_{1j}(z_{2j} + \alpha_{2j}) \tag{9}$$

We design the first distributed virtual controller α_{2j} as:

$$\alpha_{2j} = -\sum_{l \in \mathcal{N}_j} a_{jl}(z_{1j} - z_{1l}) \tag{10}$$

Note that a_{jl} represents the weighted adjacency between the neighbouring agents, and all the a_{jl} hereinafter are assumed to be 1. $\mathcal{N}_j$ denotes the neighbour set of the j-th agent and no global information states are included in α_{2j}. Information in communication networks only travels directly between immediate neighbours in the graph. Nevertheless, if the graph is connected, then this locally transmitted information ultimately travels to every agent in the graph.

With the aid of eqn. (10), (7) can be written as:

$$\dot{z}_{1j} = -\sum_{l \in \mathcal{N}_j} (z_{1j} - z_{1l}) + z_{2j} \tag{11}$$

and $\dot{V}_1$ can be written as:

$$\dot{V}_1 = -z_{1*}^T L z_{1*} + \sum_{j=1}^{m} z_{1j} z_{2j} \tag{12}$$

In the second step, and considering eqn.(6) and the second-order of eqn.(2), it is possible to obtain:

$$\begin{aligned} \dot{z}_{2j} &= x_{3j} - \dot{\alpha}_{2j} \\ &= z_{3j} + \alpha_{3j} - \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} - \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \end{aligned} \tag{13}$$

Remark 1. *α_{3j} is treated as a virtual controller for a higher-order subsystem which would be designed to guarantee the consensus of the first-order and the second-order subsystems for the multiple high-order systems. This is to say that the virtual controller α_{3j} is to be designed such that $\lim_{t\to\infty}(z_{1j} - z_{1l}) = 0$ and $\lim_{t\to\infty}(z_{2j} - z_{2l}) = 0$ for $1 \le j, l \le m$.*

Hence, we choose the second Lyapunov function candidate V_2 as:

$$V_2 = V_1 + \frac{1}{2} z_{2*}^T z_{2*} \tag{14}$$

where $z_{2*} = [z_{21}, z_{22}, \ldots, z_{2m}]^T$. Taking the time-derivative of V_2 with respect to (12) and (13), we can get:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \sum_{j=1}^{m} z_{2j}^T \dot{z}_{2j} \\ &= -z_{1*}^T L z_{1*} + \sum_{j=1}^{m} z_{1j} z_{2j} + \sum_{j=1}^{m} z_{2j} \Bigg[z_{3j} + \alpha_{3j} \\ &\quad - \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} - \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \Bigg] \end{aligned} \tag{15}$$

In order to ensure that the time-derivative of the Lyapunov function V_2 is negative definite, an appropriate α_{3j} should be designed. We design the distributed virtual controller α_{3j} as:

$$\alpha_{3j} = -z_{1j} - c_{2j} z_{2j} + \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} + \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \tag{16}$$

where c_{2j} is the design parameter satisfying $c_{2j} > 0$. Please note that α_{3j} only contains its own state information and its neighbours' information without using any global information generally.

Remark 2. *Note that the two items $-\frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j}$ and $-\sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l}$ in eqn.(15) are directly cancelled by the design of α_{3j}. Furthermore, the item $-z_{1j}$ in α_{3j} is designed to make sure that $\sum_{j=1}^{m} z_{1j} z_{2j}$ in eqn.(15) can be eliminated. In addition, the item $-c_{2j} z_{2j}$ in eqn.(16) is designed to ensure the negative definite of eqn.(15). The item $\sum_{j=1}^{m} z_{2j} z_{3j}$ in eqn.(17) will be handled in the third step by choosing an appropriate virtual controller α_{4j}.*

Therefore, by substituting (16) into (15), $\dot{V}_2$ can be rewritten as follows:

$$\dot{V}_2 = -z_{1*}^T L z_{1*} - z_{2*}^T \mathrm{diag}(c_{2*}) z_{2*} + \sum_{j=1}^{m} z_{2j} z_{3j} \tag{17}$$

where $c_{2*} = [c_{21}, c_{22}, \ldots, c_{2m}]^T$.

In step i, $1 \leq i \leq n-1$. Following the design procedures similar to the first and second steps, it is possible to obtain:

$$\begin{aligned}\dot{z}_{ij} &= x_{(i+1)j} - \dot{\alpha}_{ij} \\ &= z_{(i+1)j} + \alpha_{(i+1)j} - \sum_{k=1}^{i-1} \frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad - \sum_{k=1}^{i-1} \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l} \end{aligned} \tag{18}$$

In (18), the virtual controller $\alpha_{(i+1)j}$ which can guarantee the consensus of the multiple i-rank $(1 < i < n-1)$ subsystems should be designed such that $\lim_{t\to\infty}(z_{kj} - z_{kl}) = 0$ for $1 \leq j, l \leq m$ and $1 \leq k \leq n-1$, with the aid of the Lyapunov function:

$$V_i = V_{i-1} + \frac{1}{2} z_{i*}^T z_{i*} \tag{19}$$

Note that V_{i-1} can be designed in the $i-1$ step by the recursive method. Taking the time-derivative of V_i by considering V_{i-1} in step i and (18), we can get:

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + \sum_{j=1}^{m} z_{ij}\dot{z}_{ij} \\ &= -z_{1*}^T L z_{1*} - \sum_{j=2}^{i-1} z_{j*}^T \mathrm{diag}(c_{j*}) z_{(i-1)*} \\ &\quad + \sum_{j=1}^{m} z_{(i-1)j} z_{ij} + \sum_{j=1}^{m} z_{ij} \left[- \sum_{k=1}^{i-1} \frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \right. \\ &\quad \left. + z_{(i+1)j} + \alpha_{(i+1)j} - \sum_{k=1}^{i-1} \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l} \right] \end{aligned} \tag{20}$$

Choose the virtual controller $\alpha_{(i+1)j}$ as:

$$\begin{aligned}\alpha_{(i+1)j} &= -z_{(i-1)j} - c_{ij} z_{ij} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad + \sum_{k=1}^{i-1} \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l} \end{aligned} \tag{21}$$

where c_{ij} is the design parameter, and satisfy $c_{ij} > 0$. Substituting (21) into $\dot{V}_i$, we obtain:

$$\dot{V}_i = -z_{1*}^T L z_{1*} - \sum_{j=2}^{i} z_{i*}^T \mathrm{diag}(c_{j*}) z_{j*} + \sum_{j=1}^{m} z_{ij} z_{(i+1)j} \tag{22}$$

where $c_{i*} = [c_{i1}, c_{i2}, \ldots, c_{im}]^T$.

In the last step, since the parameter θ_j is unknown, the parameter adaptive law should be designed as well. Differentiating $z_{nj} = x_{nj} - \alpha_{nj}$, it is possible to obtain:

$$\begin{aligned}\dot{z}_{nj} &= u_j + \phi_j^T \theta_j - \dot{\alpha}_{nj} \\ &= u_j - \sum_{k=1}^{n-1} \frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1)j} - \sum_{k=1}^{n-1} \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1)l} \\ &\quad + \phi_j^T \hat{\theta}_j + \phi_j^T \tilde{\theta}_j \end{aligned} \tag{23}$$

where $\hat{\theta}_j$ is the estimate of θ_j and $\tilde{\theta}_j = \theta_j - \hat{\theta}_j$ is the estimate error.

It is worth noting that in (23) the actual control input u_j is finally explicitly included, so u_j not only causes all the ranks of the multiple high-order nonlinear systems to reach consensus but also deals with parameter uncertainties by using the Lyapunov stability analysis method.

We design the actual controller u_j from (23) such that $\lim_{t\to\infty}(z_{kj} - z_{kl}) = 0$, for $1 \leq j, l \leq m$ and $1 \leq k \leq n$, with the aid of the Lyapunov function candidate V_n as follows:

$$V_n = V_{n-1} + \frac{1}{2} z_{n*}^T z_{n*} + \frac{1}{2} \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \tilde{\theta}_j \tag{24}$$

Taking the time-derivative of V_n with respect to (22) and (23), we obtain:

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \sum_{j=1}^{m} z_{nj}\dot{z}_{nj} + \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \dot{\tilde{\theta}}_j \\ &= -z_{1*}^T L z_{1*} - \sum_{j=2}^{n-1} z_{j*}^T \mathrm{diag}(c_{j*}) z_{j*} + \sum_{j=1}^{m} z_{(n-1)j} z_{nj} \\ &\quad + \sum_{j=1}^{m} z_{nj} \left[u_j + \phi_j^T \hat{\theta}_j + \phi_j^T \tilde{\theta}_j - \sum_{k=1}^{n-1} \frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1)j} \right. \\ &\quad \left. - \sum_{k=1}^{n-1} \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1)l} \right] + \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \dot{\tilde{\theta}}_j \end{aligned} \tag{25}$$

We choose the adaptation law:

$$\dot{\hat{\theta}}_j = \Gamma_j z_{nj} \phi_j \tag{26}$$

where Γ_j is a positive definite matrix. Note that z_{nj} only contains the local information. In addition, the distributed control law is:

$$\begin{aligned}u_j &= -z_{(n-1)j} - c_{nj} z_{nj} + \sum_{k=1}^{n-1} \frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad + \sum_{k=1}^{n-1} \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1)l} - \phi_j^T \hat{\theta}_j \end{aligned} \tag{27}$$

where c_{nj} is the design parameter satisfying $c_{nj} > 0$. Note that u_j is designed relying on all the values of α_{ij}, where $1 \leq i \leq n$.

Using (26) and (27) for (25), we can derive:

$$\begin{aligned}\dot{V}_n &= -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \mathrm{diag}(c_{i*}) z_{i*} \\ &\quad + \sum_{j=1}^{m} \tilde{\theta}_j^T (z_{nj}\phi_j - \Gamma_j^{-1} \dot{\hat{\theta}}_j) \\ &= -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \mathrm{diag}(c_{i*}) z_{i*} \leq 0 \end{aligned} \tag{28}$$

Based on the above recursive backstepping analysis, we can obtain the following theorem.

Theorem 1. Consider the multiple nonlinear systems described by (2)-(3), when the communication topology of the systems is fixed and connected, and choose the control law (27) and the adaptation law (26) for system j, where $1 \leq j \leq m$, then it guarantees that the control objective (4) holds and that $\hat{\theta}_j$ is bounded - that is, the consensus of high-order nonlinear uncertain systems can be reached. In the control laws, all the required information is local. Moreover, each agent needs to know the total states of its neighbours in order to be able to implement the control law.

Proof. Given the above design procedure, define the Lyapunov function candidate as (24), then we get (28). Therefore, it follows that $z_{i*} \in \mathcal{L}^\infty$, $\tilde{\theta}_j \in \mathcal{L}^\infty$ and $\hat{\theta}_j$ is bounded according to the boundedness of θ_j. From (5), (6) and (10), we get x_{1j}, and α_{2j} and x_{2j} are bounded; furthermore, α_{3j} is bounded from (16), and following this procedure, we claim that u_j is bounded. Using the above arguments, it follows that $\dot{z}_{i*}$, $\dot{\hat{\theta}}_j$ are all bounded from (11), (13), (18), (23) and (26), and the definition of ϕ_j and Γ_j. By differentiating (28), we can see that $\ddot{V}$ is bounded, which means that $\dot{V}$ is uniformly continuous. Hence, using Barbalat's lemma [20], it follows that $\dot{V} \to 0$ as $t \to \infty$, i.e., $\lim_{t\to\infty} z_{1*}^T L z_{1*} = 0$ and $\lim_{t\to\infty} z_{l*} = \mathbf{0}_m$ for $2 \leq l \leq m$. Using $\lim_{t\to\infty} z_{2*} = \mathbf{0}_m$, (7) becomes $\dot{z}_{1j} = -\sum_{l\in\mathcal{N}_j} a_{jl}(z_{1j} - z_{1l})$, which implies that $\dot{x}_{1j} = -\sum_{l\in\mathcal{N}_j} a_{jl}(x_{1j} - x_{1l})$; thus, the consensus is reached by Lemma 2.10 in [16], i.e., for all $x_{1j}(0)$ and all $i, j = 1, \ldots, m$, $|x_{1i} - x_{1j}| \to 0$ as $t \to \infty$. We then obtain $\lim_{t\to\infty} Lx_{1*} = \mathbf{0}_m$ and $\lim_{t\to\infty} x_{1*} = a\mathbf{1}_m$ for some $a \in \mathbf{R}$ according to $L\mathbf{1}_m = \mathbf{0}_m$, where $\mathbf{1}_m$ and $\mathbf{0}_m$ denote the $m \times 1$ column vector of all ones and zeros. Now, denoting by $\bar{x}_1 = \frac{1}{n}\sum_{j=1}^m x_{1j}$ the average of the first-order states, we get $\dot{\bar{x}}_1 = \frac{1}{n}\sum_{j=1}^m \dot{x}_{1j} = -\frac{1}{n}\mathbf{1}_m^T L x_{1*} = 0$, so that $\dot{\bar{x}}_1 = \frac{1}{n}\sum_{j=1}^m x_{1j}(0)$, which means $a = \frac{1}{n}\sum_{j=1}^m x_{1j}(0)$, and the average consensus of the first-order states has been reached. When $\lim_{t\to\infty}(x_{1i} - x_{1j}) = 0$, we get $\alpha_{2j} \to 0$ as $t \to \infty$; thus, $x_{2j} \to 0$ will hold from (6) and (10). Following this step, we can further obtain that $x_{ij} \to 0$ as $t \to \infty$ for $i = 3, \ldots, n$.

Remark 3. *By considering the structural characteristics of the system, the main idea of our proposed method is to break a huge consensus problem with the multiple high-order nonlinear systems into a sequence of recursive design problems with lower-order multiple subsystems based on the backstepping frameworks. In each step of the design procedure, only local information is used to design the virtual controller, which makes it more difficult to find the appropriate controllers, but consequently the resulting actual controller and the parameter adaptive law can be obtained in a distributed manner, which overcomes the main drawbacks of the ordinary backstepping methods in which global state information must be used.*

Remark 4. *If unknown but bounded disturbances d_j are taken into account in each subsystem, a similar analysis to that of Theorem 1 can be performed:*

$$\dot{V}_n = -z_{1*}^T L z_{1*} - \sum_{i=1}^{n-1} z_{i*}^T diag(c_{i*}) z_{i*} + \sum_{j=1}^{m} z_{nj} d_j \quad (29)$$

The disturbances can be rejected by adding a robust compensation term $u_{cj} = \bar{d}_j \mathrm{sgn}(z_{nj})$ to the control law (27), where $\bar{d}_j > 0$ is the upper bound of d_j and 'sgn' is the sign function; thus, the same result can be obtained as for Theorem 1. Since the use of the sign function may cause undesirable control chatting, we replace it by a saturation function $\mathrm{sat}(z_{nj})$, where:

$$\mathrm{sat}(z_{nj}) = \begin{cases} \mathrm{sgn}(z_{nj}), & \text{if} |z_{nj}/\varepsilon_j| \geq 1 \\ z_{nj}/\varepsilon_j, & \text{if} |z_{nj}/\varepsilon_j| < 1 \end{cases} \quad (30)$$

$\varepsilon_j > 0$ denotes the boundary layers. Notice that when the saturation functions are used, the system errors can only be guaranteed to converge to the bounded layers with corresponding small tracking errors rather than zero, but the practical advantages may be significant.

Remark 5. *In the current paper, it is assumed that the communication topology is bi-directional. If the communication graph is directed and contains a spanning tree, then all the eigenvalues of the weighted Laplacian L have a non-negative real part [21]. Thus, similar consensus algorithms can be extended to a directed graph - the proof is similar to the proof of Theorem 1 and we omit the detailed steps due to space limitations.*

4. Simulation

In this section, some simulation experiments are presented to verify the theoretical analysis. In order to show the advantage of the proposed control law over those in the existing literature (in which only multiple three-order nonlinear systems with a chained form are commonly applied for simulations), in this paper, multiple uncertain nonlinear systems with four-orders are applied. The models of the systems are defined by:

$$\begin{aligned}
\dot{x}_{1j} &= x_{2j} \\
\dot{x}_{2j} &= x_{3j} \\
\dot{x}_{3j} &= x_{4j} \\
\dot{x}_{41} &= u_1 + sin(x_{11})\theta_1 \\
\dot{x}_{42} &= u_2 + [x_{12} + sin(x_{22})]\theta_2 \\
\dot{x}_{43} &= u_3 + x_{13}\theta_3 \\
\dot{x}_{44} &= u_4 + x_{14}\theta_4 \\
\dot{x}_{45} &= u_5 + sin(x_{15})\theta_5 \\
\dot{x}_{46} &= u_6 + sin(x_{16})\theta_6 \\
\dot{x}_{47} &= u_7 + x_{17}\theta_7 \\
\dot{x}_{48} &= u_8 + x_{18}\theta_8 \\
\dot{x}_{49} &= u_9 + x_{19}\theta_9 \\
\dot{x}_{4,10} &= u_{10} + x_{1,10}\theta_{10} \\
\dot{x}_{4,11} &= u_{11} + sin(x_{1,11})\theta_{11}
\end{aligned}$$

where $1 \leq j \leq 11$ and θ_j are unknown parameters.

Consider an 11-node undirected graph as described in Figure 1. According to the communication network $\mathcal{G}$, each individual agent can only exchange information

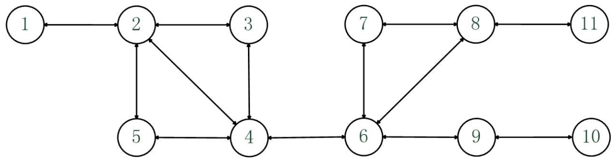

Figure 1. Communication network $\mathcal{G}$ of the multi-agent system

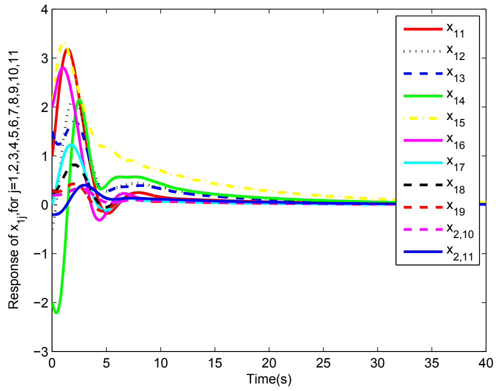

Figure 2. Response of x_{1j} for $1 \leq j \leq 11$

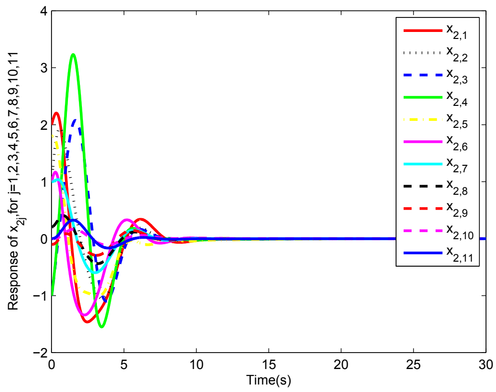

Figure 3. Response of x_{2j} for $1 \leq j \leq 11$

with its neighbouring agents. In this case, the unknown parameters are assumed to be $\theta_1 = 1, \theta_2 = 1, \theta_3 = 3, \theta_4 = 4, \theta_5 = 2, \theta_6 = 1, \theta_7 = 1, \theta_8 = 1, \theta_9 = 1, \theta_{10} = 1, \theta_{11} = 1.$. Note that the communication graph $\mathcal{G}$ satisfies Assumption 1. Furthermore, the corresponding adjacent weights between agents are assumed to be 1, and all the others are 0. The initial conditions of the systems can be chosen as: $x_{11} = 1.1, x_{21} = 2, x_{31} = 1, x_{41} = -0.5, x_{12} = -0.5, x_{22} = 1, x_{32} = 3, x_{42} = -1, x_{13} = -1.4, x_{23} = -1, x_{33} = 2, x_{43} = 1, x_{14} = -2.1, x_{24} = -1, x_{34} = 2, x_{44} = 1, x_{15} = 1.9, x_{25} = 1.8, x_{35} = 0, x_{45} = -0.5, x_{16} = 1.7, x_{26} = 1, x_{36} = 0, x_{46} = 1, x_{17} = -0.3, x_{27} = 1, x_{37} = 0, x_{47} = 1, x_{19} = 0.3, x_{29} = -0.1, x_{39} = 0, x_{49} = 0.01, x_{1,10} = -0.2, x_{2,10} = 0, x_{3,10} = 0.01, x_{4,10} = 0, x_{1,11} = -0.2, x_{2,11} = 0, x_{3,11} = 0, x_{4,11} = 0.5, \hat{\theta}_1 = 1, \hat{\theta}_2 = 1, \hat{\theta}_3 = 1, \hat{\theta}_4 = -5, \hat{\theta}_5 = -6, \hat{\theta}_6 = 1, \hat{\theta}_7 = 1, \hat{\theta}_8 = 1, \hat{\theta}_9 = 1, \hat{\theta}_{10} = 1, \hat{\theta}_{11} = 1$. The consensus control laws can be obtained by Theorem 1 in which appropriate values are designed for the control parameters Γ_j and $c_{ij}, 1 \leq j \leq 11, 1 \leq i \leq 4$. The response values of the states and the controller will be displayed as follows:

In this simulation example, as shown in Figure 1, agents four and six are the key nodes in the communication network, both of which have four neighbouring agents. Therefore, the two agents' states (as can be seen by the green and pink lines in Figure 2 to Figure 5) and the control laws (the green and pink lines in Figure 6 to Figure 7) change more dramatically than the other

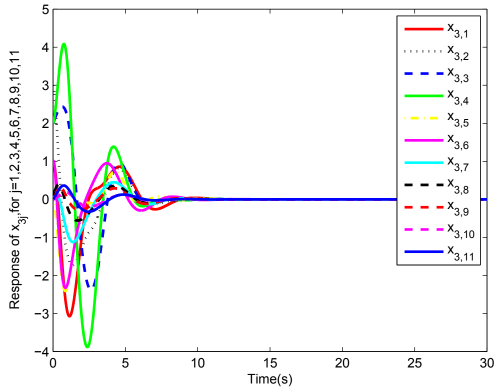

Figure 4. Response of x_{3j} for $1 \leq j \leq 11$

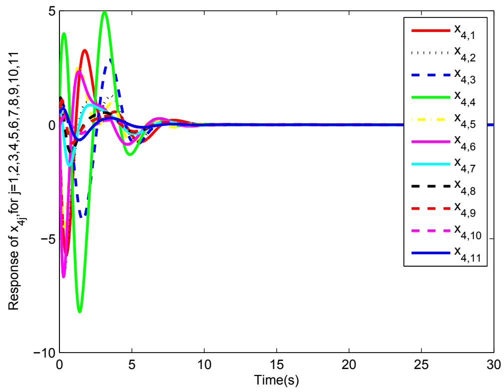

Figure 5. Response of x_{4j} for $1 \leq j \leq 11$

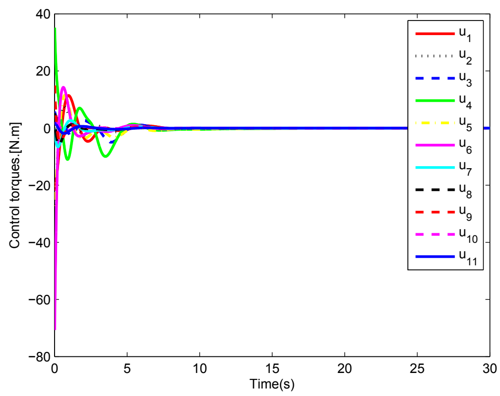

Figure 6. Response of u_j for $1 \leq j \leq 11$

nodes. Figures 2-5 show the evolution of the states. Clearly, the position states of all 11 agents ultimately reach consensus. It can be seen from Figure 6 that the control input u_j of each system converges to zero as the consensus is achieved. Figure 7 shows the estimation of the unknown parameters $\theta_j, 1 \leq j \leq 11$ by the adaptive law in (31) and the values of them converge to $[2.5542, 2.4561, 2.7336, 0.9250, 7.5837, 6.3846, 0.6568, 0.6104, 0.2174, 1.1779, 1.2443]^T$. We proved the boundedness of the estimates and have illustrated the efficiency of the adaptation law in the simulation, which can be seen from Figure 7. The first-order states converge to the average of

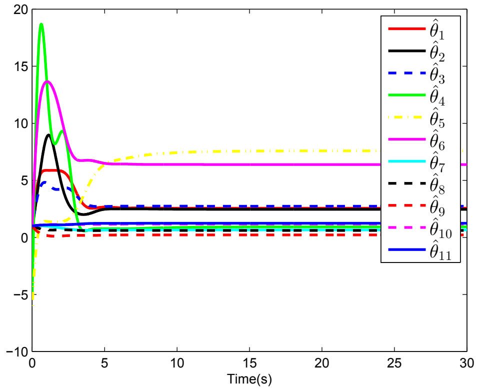

Figure 7. Response of $\hat{\theta}_j$ for $1 \leq j \leq 11$

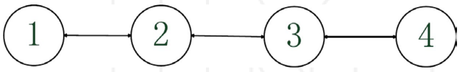

Figure 8. A linear communication network for the multi-agent system

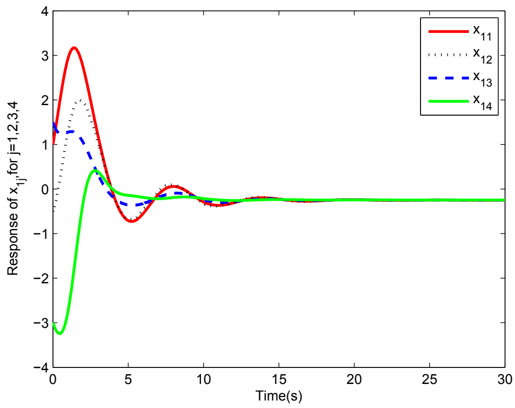

Figure 9. Response of x_{1j} for $1 \leq j \leq 4$, according to Figure 8

their initial values, while all the other states converge to zero.

A linear network (as shown in Figure 8) is the one which induces the slowest response, because the information needs to propagate from one side of the network to the other. In this case, the simulation (as shown in Figure 9) can also verify the same result. It can be seen that the control objective (4) holds. Therefore, the distributed consensus control laws in Theorem 1 are effective.

5. Conclusion

This paper discussed the consensus control problem of a type of multiple high-order nonlinear system with uncertainties. We proposed a distributed backstepping design method that interlaces the elaborate design of a suite of Lyapunov functions with the design of virtual feedback control. The controller design problem for the high-order multiple systems was broken into a sequence of design problems for lower-order subsystems. More specifically, a distributed consensus control approach under a fixed and undirected communication graph was devised and distributed backstepping techniques were utilized to construct the virtual intermediate control functions. Finally, simulations with multiple four-order uncertain nonlinear systems were provided to show consistency with the theoretical results. The design techniques herein can be applied to a wide class of multiple high-order nonlinear systems with uncertainties. Since time-varying and a switching communication topology are commonly applied in the real-world, research into these problems would be very interesting and meaningful. Further work includes extending the result to cases when there exist communication delays between systems, where the states are not available and when the high-order nonlinear system has uncertainties at each step.

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