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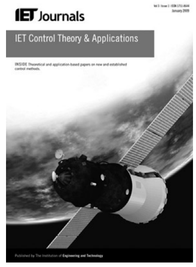

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Networked predictive control for systems with unknown or partially known delay

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Abstract: This study is concerned with the problem of controller design for linear systems with unknown or partially known communication delay. A new networked predictive control scheme is proposed to deal with the unknown or partially known communication delay. The closed-loop system is modelled as a switched system under constraint switching taking an important property of communication delay into account. Sufficient stability conditions are derived using switched Lyapunov function approach. On the basis of stability conditions, the method of designing the controller gain matrix and some important parameters in the control scheme is investigated. Finally, a numerical example is given to confirm the effectiveness of the proposed method.

1 Introduction

Networked control systems (NCSs) whose components are connected with some form of communication networks have received much attention in recent years. NCSs have many practical advantages such as reduced wiring and power, low cost, ease of diagnosis and maintenance, and high flexibility of operations [1–3]. Along with advantages, some disadvantages such as network-induced delay, data dropout, data disorder and quantised error are also introduced.

To deal with these disadvantages, many methodologies have been proposed such as time-delay system method, jump system method, switched system method, impulsive systems approach and so on. For example, in [4–6], the NCS was modelled as a system with time-varying delay and delay-dependent stability conditions were obtained. In [7], both the sensor-to-controller and the controller-to-actuator network-induced delay were modelled as Markov chains and a two-mode dependent controller was designed based on necessary and sufficient stochastic stability conditions. In [8], NCSs with time-varying transmission intervals and transmission constraints were modelled as discrete-time switched linear uncertain systems and stability conditions for the closed-loop system were derived using a convex overapproximation method. In [9], NCSs were modelled as linear impulsive systems and state-feedback controller were designed by employing Lyapunov functions with discontinuity. Other methodologies can be found in some good survey papers [10–13] and references therein.

It should be pointed out that the network-induced delay is usually assumed to be measurable in existing publications. Using the so-called 'time-stamp' approach, the value of the network-induced delay can be obtained. However, it is not an easy task since all the components in the system are needed to be synchronised. To the best of authors' knowledge, there are few results concerning the controller design problem for NCSs with unknown or partially known network-induced delay. By decomposing unknown network-induced delay into a fixed part and a randomly varying part, delay-dependent stability conditions were developed for the static controller design based on robust control method and Lyapunov functional approach [14]. The problem of fault detection of NCSs with unknown network-induced delay and unknown input was investigated in [15] using eigen-decomposition, adaptive evaluation and adaptive threshold. By employing Taylor series expansion, the controller design problem in the context of unknown time varying delays was transformed into a problem of stabilisation of uncertain systems with polytopic uncertainties [16].

It should be noted that the above works did not consider how to compensate for network-induced delay actively. Recently, a networked predictive control method has been proposed to actively compensate for network-induced delay [17–24]. Generally speaking, the networked predictive control method has three key steps. First, the controller node generates a series of future control signals based on the model of the plant using model predictive control or some iterative algorithms. Second, all the future control signals are packed into one packet and were sent to the actuator node. Finally, the actuator node selects an appropriate control signal from the received packet according to the network-induced delay. Both simulations and experimental results show that this method is quite effective and can obtain a similar performance as local control. However, the network-induced delay is assumed to be known in networked predictive control method. If the network-induced delay is unknown or partially known, how to design a networked predictive controller motivates this study. Part of the materials in this study can be found in [25].

In this paper, a control scheme which is robust to the unknown or partially communication delay is proposed based on the networked predictive control method. By taking an important property of communication delay into account, the closed-loop system is transformed into a switched system under constraint switching. Sufficient stability conditions are derived using switched Lyapunov function approach and presented in terms of linear matrix inequalities (LMIs). The controller gain matrix and some parameters can be obtained by solving LMIs. Finally, a numerical example is given to confirm the validity and effectiveness of the proposed method.

2 Problem formulation

Consider the following discrete-time linear system

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

$$y(k) = Cx(k) \quad (2)$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^m$ is the control input; $y(k) \in \mathbb{R}^l$ is the output vector; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{l\times n}$ are constant system matrices.

For the sake of simplicity, but without loss of generality, NCSs with random delay in the feedback channel (from sensor to controller) are considered in this paper. The following assumptions are made.

Assumption 1: The pair (A, B) is completely controllable.

Assumption 2: The network-induced delay $\tau(k)$ is assumed to vary within an interval $\tau_1 \le \tau(k) \le \tau_2$, where τ_1 and τ_2 are known positive integers.

Following traditional networked predictive control scheme [17–23], if the network-induced delay is measurable, based on the received state $x(k-\tau(k))$ at the controller node, the state predictions from time $k-\tau(k)+1$ to k can be constructed as

$$\begin{aligned} x(k-\tau(k)+1) &= Ax(k-\tau(k)) + Bu(k-\tau(k)) \\ x(k-\tau(k)+2) &= Ax(k-\tau(k)+1) + Bu(k-\tau(k)+1) \\ &= A^2x(k-\tau(k)) + ABu(k-\tau(k)) \\ &\quad + Bu(k-\tau(k)+1) \\ &\;\;\vdots \\ x(k) &= A^{\tau(k)}x(k-\tau(k)) + \sum_{i=1}^{\tau(k)} A^{i-1}Bu(k-i) \end{aligned} \quad (3)$$

From the above state predictions, the networked predictive controller is designed as

$$\begin{aligned} u(k) &= Kx(k) \\ &= KA^{\tau(k)}x(k-\tau(k)) + K\sum_{i=1}^{\tau(k)} A^{i-1}Bu(k-i) \end{aligned} \quad (4)$$

where K is the controller gain matrix.

However, the above control method needs that the network-induced delay is exactly measurable. When the network-induced delay $\tau(k)$ is unknown or partially unknown, the above control method is unavailable. Two cases of network-induced delay is considered in this paper. One is that the network-induced delay is unknown. The other is that the network-induced delay is partially known, that is, the exact value of the network-induced delay is unknown but some other information about it is known.

Case 1: For the unknown network-induced delay case, similar to [26] the following state feedback controller is proposed

$$u(k) = KA^hx(k-\tau(k)) + K\sum_{j=1}^{h} A^{j-1}Bu(k-j) \quad (5)$$

where h is the prediction step to be determined and satisfies $\tau_1 \le h \le \tau_2$.

Case 2: For the partially known network-induced delay case, the integer interval $[\tau_1, \tau_2]$ can be divided into γ subintervals $[\tau_1, d_1], [d_1+1, d_2], \ldots, [d_{\gamma-1}+1, \tau_2]$. For brief, we use $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_\gamma$ representing the above γ subintervals. For a network-induced delay $\tau(k)$, though we do not know its accurate value, we know the subinterval which it belongs to. It should be noted that Case 2 turns into Case 1 when $\gamma = 1$ and Case 2 turns into the case that the network-induced delay is known when $\gamma = \tau_2 - \tau_1 + 1$.

For each $\mathcal{S}_i$, $i = 1, 2, \ldots, \gamma$, we choose an prediction step $h_i \in \mathcal{S}_i$. If $\tau(k) \in \mathcal{S}_i$, the following state feedback controller is proposed

$$u(k) = KA^{h_i}x(k-\tau(k)) + K\sum_{j=1}^{h_i} A^{j-1}Bu(k-j) \quad (6)$$

where h_i, $i = 1, 2, \ldots, \gamma$ are the prediction step to be determined.

3 Stability analysis

In this section, stability of the closed-loop system is analysed and sufficient stability conditions are obtained using switched Lyapunov function approach.

3.1 Unknown network-induced delay

Substituting (5) into (1) yields

$$\begin{aligned} x(k+1) &= Ax(k) + BKA^hx(k-\tau(k)) \\ &\quad + BK\sum_{j=1}^{h} A^{j-1}Bu(k-j) \end{aligned} \quad (7)$$

It it clear that

$$x(k) = A^hx(k-h) + \sum_{j=1}^{h} A^{j-1}Bu(k-j) \quad (8)$$

Therefore

$$\sum_{j=1}^{h} A^{j-1}Bu(k-j) = x(k) - A^hx(k-h) \quad (9)$$

From (7) and (9), one can obtain

$$\begin{aligned} x(k+1) &= (A+BK)x(k) + BKA^hx(k-\tau(k)) \\ &\quad - BKA^hx(k-h) \end{aligned} \quad (10)$$

Remark 1: From (10), it is easy to see that if $\tau(k) \equiv h$, the closed-loop system can be described as $x(k+1) = (A + BK)x(k)$. It means that the network-induced delay is completely compensated.

The closed-loop system (10) can be seen as a system with two time-delays. One is constant and the other one is time-varying. Therefore we can use Lyapunov–Krasovskii functional method [26–28] to analyse its stability. However, under the assumption that the controller always use the most latest data, if $x(k - \tau(k))$ is available at step k but there is no new information arriving at step $k+1$, then $x(k - \tau(k))$ is at least available for controller design. So the network-induced delay will increase at most by 1 each step, that is, $\tau(k+1) \le \tau(k) + 1$. Methods in [26–28] only consider $\tau_1 \le \tau(k) \le \tau_2$ but ignore this important property of network-induced delay $\tau(k)$. Taking this property into account, we transform the closed-loop system (10) into a switched system under constraint switching.

Define a new vector

$$z(k) = \left[x^T(k) \;\; x^T(k-1) \;\; \cdots \;\; x^T(k-\tau_2+1) \;\; x^T(k-\tau_2)\right]^{\mathrm{T}}$$

and system (10) can be rewritten as the following switched system

$$z(k+1) = \Upsilon_{\sigma(k),h} z(k) \tag{11}$$

where $\sigma(k) \in \mathcal{I}$ with $\mathcal{I} = \{\tau_1, \cdots, \tau_2\}$, is a piecewise constant function and denotes the active mode.

$$\Upsilon_{i,h} = \begin{bmatrix} 0 & \overbrace{0 \cdots 0}^{i-1} & BKA^h & 0 & \cdots & 0 \\ & & & & & 0 \\ & & 0_{\tau_2 n \times \tau_2 n} & & & \vdots \\ & & & & & 0 \end{bmatrix} + \begin{bmatrix} A + BK & \overbrace{0 \cdots 0}^{h-1} & -BKA^h & 0 & \cdots & 0 \\ & & & & & 0 \\ & & I_{\tau_2 n \times \tau_2 n} & & & \vdots \\ & & & & & 0 \end{bmatrix}, \quad i \in \mathcal{I}$$

Since the network-induced delay satisfies $\tau(k+1) \le \tau(k) + 1$, it is clear that $\sigma(k+1) \le \sigma(k) + 1$. Therefore the switched system (11) is under constraint switching but not arbitrary switching.

Recently, switched systems have received great developments. Rich results for switched systems can be used to analyze networked control system [29–31]. Using switched Lyapunov function approach [32], a sufficient stability condition for system (11) is developed as the following theorem.

Theorem 1: For given controller gain matrix K and prediction step h, system (11) is asymptotically stable if there exist matrices $P_i > 0$ and any matrices G_i such that

$$\begin{bmatrix} -P_i & * \\ G_i \Upsilon_{i,h} & P_j - G_i - G_i^T \end{bmatrix} < 0, \quad \forall (i, j) \in \mathcal{I} \times \mathcal{I} \quad \text{and} \quad j \le i+1 \tag{12}$$

Proof: Define a indicator function $\alpha(k) = [\alpha_{\tau_1}(k), \ldots, \alpha_{\tau_2}(k)]^{\mathrm{T}}$ with

$$\alpha_i(k) = \begin{cases} 1, & \sigma(k) = i \\ 0, & \text{otherwise} \end{cases}$$

Similar to [32, 33], choose a switched Lyapunov function

$$\begin{aligned} V(k, z(k)) &= z^T(k) P(\alpha(k)) z(k) \\ &= z^T(k) \left(\sum_{i=\tau_1}^{\tau_2} \alpha_i(k) P_i \right) z(k) \end{aligned} \tag{13}$$

One can obtain that

$$\begin{aligned} \Delta V &= V(k+1, z(k+1)) - V(k, z(k)) \\ &= z^T(k+1) P(\alpha(k+1)) z(k+1) - z^T(k) P(\alpha(k)) z(k) \\ &= z^T(k+1) P_j z(k+1) - z^T(k) P_i z(k) \end{aligned} \tag{14}$$

It is clear that the following equation holds for all $(i, j) \in \mathcal{I} \times \mathcal{I}$ and $j \le i+1$

$$0 = 2z^T(k+1) G_i \left[-z(k+1) + \Upsilon_{i,h} z(k)\right] \tag{15}$$

Adding both sides of (15) to both sides of (14) yields

$$\begin{aligned} \Delta V &= z^T(k+1)(P_j - G_i - G_i^T) z(k+1) - z^T(k) P_i z(k) \\ &\quad + 2z^T(k+1) G_i \Upsilon_{i,h} z(k) \end{aligned} \tag{16}$$

So if (12) holds, then $\Delta V < 0$ which means that system (11) is asymptotically stable. □

Definition 1: A switched system $x(k+1) = A_{\sigma(k)} x(k)$ is said to be exponentially stable with a decay rate $\lambda > 1$, if its solution $x(k)$ satisfies $x(k) \le \kappa \lambda^{-k} \|x(0)\|$, $\forall k \ge 0$, where $\kappa > 0$ is a constant.

From [33], we can see that if the system (11) is asymptotically stable, it is also exponentially stable. A decay rate can be estimated by the following theorem.

Theorem 2: For given controller gain matrix K and prediction step h, system (11) is exponentially stable with a decay rate $\lambda = [1/(\sqrt{1-\nu})]$, if there exist matrices $\eta I < P_i < I$, any matrices G_i and scalars $\nu > 0$, $\eta > 0$, such that

$$\begin{bmatrix} -P_i + \nu I & * \\ G_i \Upsilon_{i,h} & P_j - G_i - G_i^T \end{bmatrix} < 0, \quad \forall (i, j) \in \mathcal{I} \times \mathcal{I} \quad \text{and} \quad j \le i+1 \tag{17}$$

Proof: Following the similar line as in [33], we can see the Lyapunov function (13) satisfies

$$\eta \|z(k)\|^2 \le V(k, z(k)) \le \rho \|z(k)\|^2 \tag{18}$$

where η and ρ are finite positive constants.

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If (12) holds, there will exist a sufficient small $\nu > 0$ such that

$$V(k+1, z(k+1)) - V(k, z(k)) \leq -\nu \|z(k)\|^2 \tag{19}$$

From (18), we can obtain $-\|z(k)\|^2 \leq -V(k, z(k))/\rho$. Therefore

$$V(k+1, z(k+1)) \leq (1 - \nu/\rho)V(k, z(k)) \tag{20}$$

Let $\lambda = [1/(\sqrt{1-\nu/\rho})]$, we can obtain

$$V(k, z(k)) \leq \lambda^{-2k} V(0, z(0)) \tag{21}$$

From (18), we can obtain

$$\eta \|z(k)\|^2 \leq \rho \lambda^{-2k} \|z(0)\|^2 \tag{22}$$

That is

$$\|z(k)\| \leq \sqrt{\rho/\eta}\lambda^{-k} \|z(0)\| \tag{23}$$

Equations (18) and (19) can be rewritten as

$$\eta/\rho I < P_i/\rho < I \tag{24}$$

$$\begin{bmatrix} -P_i + \nu I & * \\ G_i \Upsilon_{i,h} & P_j - G_i - G_i^T \end{bmatrix} < 0 \tag{25}$$

Pre- and post-multiplying both side of (25) with

$$\begin{bmatrix} 1/\sqrt{\rho} & 0 \\ 0 & 1/\sqrt{\rho} \end{bmatrix}$$

and its transpose yields

$$\begin{bmatrix} -P_i/\rho + \nu/\rho I & * \\ G_i/\rho \Upsilon_{i,h} & P_j/\rho - G_i/\rho - G_i^T/\rho \end{bmatrix} < 0 \tag{26}$$

Define new variables $\nu = \nu/\rho$, $\eta = \eta/\rho$, $G_i = G_i/\rho$, $P_i = P_i/\rho$ and (17) and $\eta I < P_i < I$ can be obtained. The proof is completed. □

3.2 Partially known network-induced delay

Substituting (6) into (1) yields

$$x(k+1) = Ax(k) + BKA^{h_i} x(k - \tau(k)) + BK \sum_{j=1}^{h_i} A^{j-1} Bu(k-j), \quad \tau(k) \in \mathcal{S}_i \tag{27}$$

Similarly, the following equation holds for $i = 1, 2, \ldots, \gamma$

$$\sum_{j=1}^{h_i} A^{j-1} Bu(k-j) = x(k) - A^{h_i} x(k - h_i) \tag{28}$$

From (27) and (28), one can obtain

$$x(k+1) = (A + BK)x(k) + BKA^{h_i} x(k - \tau(k)) - BKA^{h_i} x(k - h_i), \quad \tau(k) \in \mathcal{S}_i \tag{29}$$

The closed-loop system (29) can be seen as a switched system with two time-delays whose sub-system is described by (29). Although we can use Lyapunov–Krasovskii functional method to analyse its stability, an important property of network-induced delay $\tau(k)$, that is, $\tau(k+1) \leq \tau(k) + 1$ will be ignored. Therefore we transform closed-loop system (29) into the following switched system under constraint switching

$$z(k+1) = \Upsilon_{\sigma(k), h_1, \ldots, h_\gamma} z(k) \tag{30}$$

where $\sigma(k) \in \mathcal{I}$ with $\mathcal{I} = \{\tau_1, \tau_1 + 1, \ldots, \tau_2\}$ denotes the active mode

$$\Upsilon_{j,h_1,\ldots,h_\gamma} = \begin{bmatrix} 0 & \overbrace{0 \cdots 0}^{j-1} & BKA^{\mu_j} & 0 & \cdots & 0 \\ & & & & & 0 \\ & & 0_{\tau_2 n \times \tau_2 n} & & & \vdots \\ & & & & & 0 \end{bmatrix} + \begin{bmatrix} A + BK & \overbrace{0 \cdots 0}^{\mu_j - 1} & -BKA^{\mu_j} & 0 & \cdots & 0 \\ & & & & & 0 \\ & & I_{\tau_2 n \times \tau_2 n} & & & \vdots \\ & & & & & 0 \end{bmatrix}, \quad j \in \mathcal{I}$$

where $\mu_j = h_i$ if $j \in \mathcal{S}_i$, $i \in \{1, 2, \ldots, \gamma\}$.

Using the switched Lyapunov function approach [32], a sufficient stability condition for system (30) is derived in the following theorem.

Theorem 3: For given controller gain matrix K and prediction step h_i, $i = 1, 2, \ldots, \gamma$, system (30) is asymptotically stable if there exist matrices $P_j > 0$ and any matrices G_j such that

$$\begin{bmatrix} -P_j & * \\ G_j \Upsilon_{j,h_1,\ldots,h_\gamma} & P_l - G_j - G_j^T \end{bmatrix} < 0, \quad \forall (j, l) \in \mathcal{I} \times \mathcal{I} \text{ and } l \leq j + 1 \tag{31}$$

Similarly, system (30) is also exponentially stable. A decay rate can be estimated by the following theorem.

Theorem 4: For given controller gain matrix K and prediction step h_i, $i = 1, 2, \ldots, \gamma$, system (11) is exponentially stable with a decay rate $\lambda = (1/\sqrt{1-\nu})]$, if there exist matrices $\eta I < P_j < I$, any matrices G_j and scalars $\nu > 0$, $\eta > 0$, such that

$$\begin{bmatrix} -P_j + \nu I & * \\ G_j \Upsilon_{j,h_1,\ldots,h_\gamma} & P_l - G_j - G_j^T \end{bmatrix} < 0, \quad \forall (j, l) \in \mathcal{I} \times \mathcal{I} \text{ and } l \leq j + 1 \tag{32}$$

4 Controller design

In this section, the design problem of the controller gain matrix K and the prediction step h or h_i, $i = 1, 2, \ldots, \gamma$ is considered. The basic idea is that appropriate controller gain matrix K and the prediction step h or h_i, $i = 1, 2, \ldots, \gamma$ should make the decay rate λ of the closed-loop system as large as possible.

Before moving on, the following lemma is introduced, which has an important role in the controller design.

Lemma 1: For a given $B \in \mathbb{R}^{n\times m}$ with

$$\operatorname{rank}(B) = m \text{ and } B = U\begin{bmatrix}\Xi\\0\end{bmatrix}V^T$$

if the matrix R has the following structure

$$R = U\begin{bmatrix}R_1 & R_2\\0 & R_3\end{bmatrix}U^T$$

then there exists a matrix $Z \in \mathbb{R}^{m\times m}$ such that $RB = BZ$, and $Z = V\Xi^{-1}R_1\Xi V^T$, where $U \in \mathbb{R}^{n\times n}$, $V \in \mathbb{R}^{m\times m}$ are orthogonal matrices, $\Xi = \operatorname{diag}(\xi_1, \xi_2, \cdots \xi_m)$, $\xi_i\ (i = 1, 2, \ldots m)$ are non-zero singular values of B, $R_1 \in \mathbb{R}^{m\times m}$, $R_2 \in \mathbb{R}^{m\times(n-m)}$ and $R_3 \in \mathbb{R}^{(n-m)\times(n-m)}$.

Proof: This lemma is a generalisation of Lemma 3 in [34]. Without loss of generality, it is assumed that $m < n$. From

$$B = U\begin{bmatrix}\Xi\\0\end{bmatrix}V^T$$

and $RB = BZ$, we can obtain that

$$RU\begin{bmatrix}\Xi\\0\end{bmatrix}V^T = U\begin{bmatrix}\Xi\\0\end{bmatrix}V^TZ$$

that is

$$U^TRU\begin{bmatrix}\Xi\\0\end{bmatrix}V^T = \begin{bmatrix}\Xi\\0\end{bmatrix}V^TZ$$

Assume

$$R = U\begin{bmatrix}R_1 & R_2\\R_4 & R_3\end{bmatrix}U^T$$

It can be obtained that

$$\begin{bmatrix}R_1\Xi V^T\\R_4\Xi V^T\end{bmatrix} = \begin{bmatrix}\Xi V^TZ\\0\end{bmatrix}$$

Therefore the above equation is solvable if $R_4 = 0$. Particularly, if R is a symmetric matrix, the above equation is solvable if $R_2 = R_4 = 0$, which is the result of Lemma 3 in [34]. In addition, it is easy to obtain that $Z = V\Xi^{-1}R_1\Xi V^T$. □

4.1 Unknown network-induced delay

On the basis of Theorem 2, a method of designing the controller gamin matrix K and the prediction step h is given in the following theorem.

Theorem 5: To make decay rate of the system (11) as large as possible, an appropriate h is given by $h^* = \arg\max\{\nu_l^{\text{opt}},\ \ l \in \mathcal{I}\}$, where ν_l^{opt} is the optimal value of the following optimisation problem for variables $\eta_l I < P_i^l < I$, $G_i^l = U\begin{bmatrix}G_{11}^l & G_{i12}^l\\0 & G_{i22}^l\end{bmatrix}U^T$, $\hat{K}_l$, $\nu_l > 0$, and $\eta_l > 0$.

Maximise ν_l
Subject to

$$\begin{bmatrix}-P_i^l + \nu_l I & *\\ G_i^l\Phi + \hat{B}\hat{K}_l\mathcal{C}_{i,l} & P_j^l - G_i^l - {G_i^l}^T\end{bmatrix} < 0, \quad \forall (i, j) \in \mathcal{I}\times\mathcal{I} \text{ and } j \le i + 1 \tag{33}$$

where

$$\Phi = \begin{bmatrix}A & 0 & 0 & \cdots & 0\\ & & & & 0\\ & & I_{\tau_2 n\times\tau_2 n} & & \vdots\\ & & & & 0\end{bmatrix}, \quad \hat{B} = \begin{bmatrix}B\\0\\\vdots\\0\end{bmatrix}$$

$$\mathcal{C}_{i,l} = \Big[I\ \ \overbrace{0\ \cdots\ 0}^{i-1}\ \ A^l\ \ 0\ \ \cdots\ \ 0\Big] + \Big[0\ \ \overbrace{0\ \cdots\ 0}^{l-1}\ \ -A^l\ \ 0\ \ \cdots\ \ 0\Big], \quad i \in \mathcal{I}$$

the corresponding feedback controller gain matrix is given by $K = V\Xi^{-1}{G_{11}^{h^*}}^{-1}\Sigma V^{\mathrm{T}}\hat{K}_{h^*}$, and Ξ, U, V are defined by

$$\hat{B} = U\begin{bmatrix}\Xi\\0\end{bmatrix}V^T$$

Proof: It is easy to see that

$$\Upsilon_{i,h} = \Phi + \hat{B}K\mathcal{C}_{i,h}, \quad i \in \mathcal{I} \tag{34}$$

From

$$G_i^l = U\begin{bmatrix}G_{11}^l & G_{i12}^l\\0 & G_{i22}^l\end{bmatrix}U^T, \quad \hat{B} = U\begin{bmatrix}\Xi\\0\end{bmatrix}V^T$$

and Lemma 1, there exists a Z^l such that

$$G_i^l\hat{B} = \hat{B}Z^l \tag{35}$$

From (17), (34) and (35), and define $\hat{K}_l = Z^lK$, and (33) can be obtained. From Lemma 1, one can obtain $Z^l = V\Xi^{-1}G_{11}^l\Xi V^{\mathrm{T}}$. Therefore $K = V\Xi^{-1}{G_{11}^l}^{-1}\Sigma V^{\mathrm{T}}\hat{K}_l$. □

4.2 Partially known network-induced delay

For partially known network-induced delay, a method of designing the controller gamin matrix K and prediction step h_i, $i = 1, 2, \ldots, \gamma$ is given as follows based on Theorem 4.

Theorem 6: To make decay rate of the system (30) as large as possible, appropriate h_i are given by $\{h_1^*, h_2^*, \ldots, h_\gamma^*\} = \arg\max\{\nu_{l_1,\ldots,l_\gamma}^{\text{opt}},\ \ l_i \in \mathcal{S}_i,\ \ i = 1, 2, \ldots, \gamma\}$, where $\nu_{l_1,\ldots,l_\gamma}^{\text{opt}}$ is the optimal value of the following optimisation problem for variables

$$\eta_{l_1,\ldots,l_\gamma}I < P_j^{l_1,\ldots,l_\gamma} < I, \quad G_j^{l_1,\ldots,l_\gamma} = U\begin{bmatrix}G_{11}^{l_1,\ldots,l_\gamma} & G_{i12}^{l_1,\ldots,l_\gamma}\\0 & G_{i22}^{l_1,\ldots,l_\gamma}\end{bmatrix}U^T,$$

$$\hat{K}_{l_1,\ldots,l_\gamma}, \quad \nu_{l_1,\ldots,l_\gamma} > 0 \text{ and } \eta_{l_1,\ldots,l_\gamma} > 0$$

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Maximise $\nu_{l_1,\ldots,l_\gamma}$
Subject to

$$\begin{bmatrix} -P_j^{l_1,\ldots,l_\gamma} + \nu_{l_1,\ldots,l_\gamma} I & * \\ G_j^{l_1,\ldots,l_\gamma}\Phi + \hat{B}\hat{K}\mathcal{C}_{j,l_1,\ldots,l_\gamma} & P_k^{l_1,\ldots,l_\gamma} - G_j^{l_1,\ldots,l_\gamma} - G_j^{l_1,\ldots,l_\gamma T} \end{bmatrix} < 0$$

$$\forall (j,k) \in \mathcal{I} \times \mathcal{I} \text{ and } k \le j+1 \tag{36}$$

where

$$\mathcal{C}_{j,l_1,\ldots,l_\gamma} = \begin{bmatrix} I & \overbrace{0 \ \cdots \ 0}^{j-1} & A^{\mu_j} & 0 & \cdots & 0 \end{bmatrix} + \begin{bmatrix} 0 & \overbrace{0 \ \cdots \ 0}^{\mu_j - 1} & -A^{\mu_j} & 0 & \cdots & 0 \end{bmatrix}, \quad j \in \mathcal{I}$$

with $\mu_j = l_i$ if $j \in \mathcal{S}_i$, $i \in \{1, 2, \ldots, \gamma\}$. The corresponding feedback controller gain matrix is given by $K = V\Xi^{-1}G_{11}^{h_1^*,\ldots,h_\gamma^* -1}\Sigma V^{\mathrm{T}}\hat{K}_{h_1^*,\ldots,h_\gamma^*}$, and Φ, $\hat{B}$, Ξ, U, V are as defined in Theorem 5.

5 Numerical example

In this section, a numerical example is given to show the effectiveness of the method developed in this paper.

Consider the following system with

$$A = \begin{bmatrix} 1.1 & 0 \\ -1.0 & -0.8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0]$$

Clearly, the above system is the open-loop unstable. The network-induced delay is assumed to be bounded by $1 \le \tau(k) \le 5$ as shown in Fig. 1. Two cases of the network-induced delay are considered.

Case 1: Unknown network-induced delay.

For this case, any information about the network-induced delay is unknown except its upper and lower bound. Using the proposed method, we can obtain an appropriate $h = 3$ and the corresponding feedback control gain matrix K is

$$K = [-0.2269 \quad 0.0769]$$

Let $x(0) = [10 \quad 0]^{\mathrm{T}}$, two simulations are conducted. One is the output of the system using the control law (5). The other is the output of the system using the state feedback controller $u(k) = Kx(k - \tau(k))$ with the same controller gain matrix K. Simulation results are given in Fig. 2. We can see that the closed-loop system is asymptotically stable using the proposed method and the control performance of the proposed method is much better than the one of the state feedback control without delay compensation. Therefore the method proposed in this paper can compensate for the unknown network-induced delay to some extent.

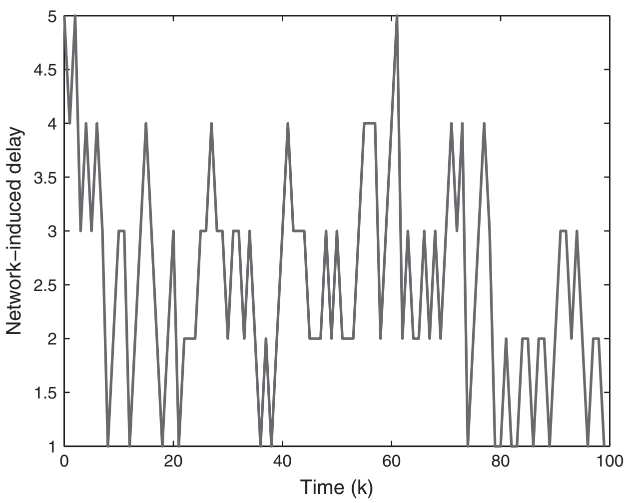

Fig. 1 *Network-induced delay*

Case 2: Partially known network-induced delay

It is assumed that the network-induced delay can be divided into two subintervals, that is, [1, 3] and [4, 5]. Using the proposed method, we can obtain appropriate $h_1 = 2$ and $h_2 = 4$, and the corresponding feedback control gain matrix K is

$$K = [-0.0999 \quad 0.2608]$$

Similarly, simulation results under different control methods are given in Fig. 3. We also can see the proposed method outperforms the state feedback control without delay compensation from Fig. 3.

Next, we will compare the results between Cases 1 and 2. For the sake of fairness, this two cases use a same $K = \begin{bmatrix} -0.2269 & 0.0769 \end{bmatrix}$. Simulation results are given in Fig. 4. We can see that Case 2 outperforms Case 1 since it uses more information about the delay.

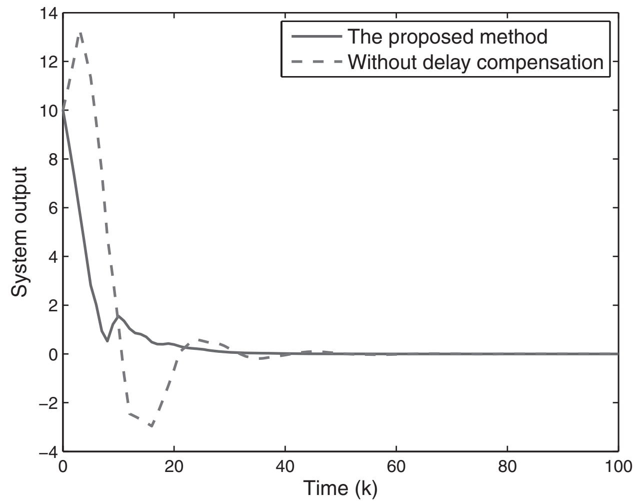

Fig. 2 *System output for Case 1 under different control methods*

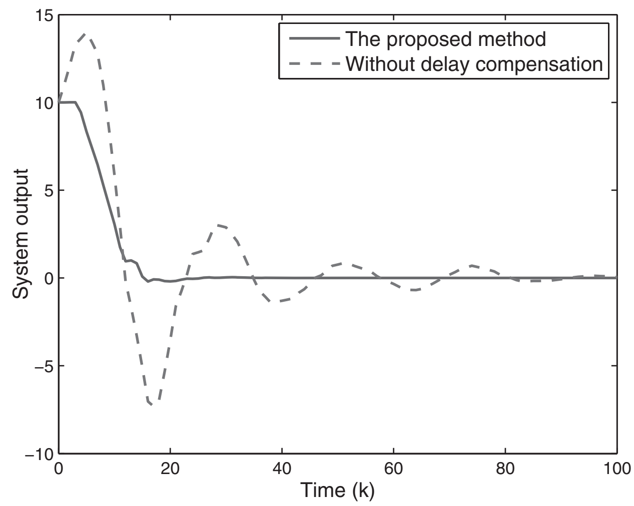

Fig. 3 *System output for Case 2 under different control methods*

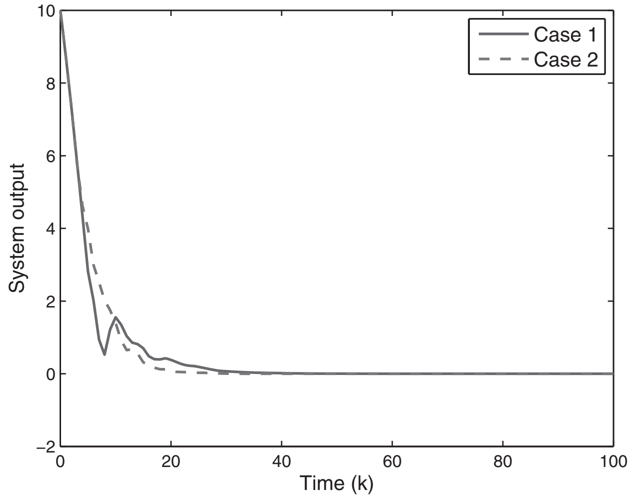

Fig. 4 *System output for Cases 1 and 2 with the same controller*

6 Conclusions

The problem of controller design for networked control systems with unknown or partially known time-varying communication delay has been investigated in this paper. A modified networked predictive control scheme has been proposed to deal with the unknown or partially known delay. The closed-loop system has been converted to a discrete-time switched system. Using switched Lyapunov functional method, sufficient stability conditions have been obtained. The controller design method has been also considered. Finally, effectiveness of the proposed method has been illustrated by a numerical example.

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