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–Brief Paper–

BOUNDED CONNECTIVITY-PRESERVING LEADER-FOLLOWER FLOCKING ALGORITHMS WITHOUT ACCELERATION MEASUREMENTS

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ABSTRACT

The problem of distributed connectivity-preserving leader-follower flocking of multiple autonomous agents with second-order dynamics is investigated. First, a new class of bounded artificial potential fields is carefully designed which could guarantee connectivity preservation, distance stabilization and collision avoidance simultaneously as the system evolves. Furthermore, in the absence of acceleration measurements of the dynamic leader, a set of distributed and bounded leader-follower flocking control protocols is derived for each follower with the aid of the combination of potential based gradient descent methods and the sliding mode control paradigms. It is shown that all followers achieve velocity consensus and collision avoidance with the dynamic leader, the underlying network remains connected for all time, and the desired stable flocking behavior is asymptotically achieved on the condition that the initial network is connected. Finally, nontrivial simulations and experiments are worked out to verify the effectiveness of the proposed control algorithms.

Key Words: Leader-follower flocking, second-order dynamics, dynamic leader, connectivity preservation, bounded control input.

I. INTRODUCTION

1.1 Previous work

Distributed flocking of networked agents has received considerable attention in recent years to solve a wide variety of spatially distributed tasks [1–3,7,8,34,38–40]. Reynolds proposed the *boids* model which consists of three heuristic rules of separation, cohesion and alignment [2]. Inspired by the Reynolds' model, many flocking algorithms were presented by a combination of velocity consensus combined with local artificial potential field (APF) [4,8,9]. As a common property, network connectedness is most commonly assumed rather than proved in the above results, which is impractical because the communication network is usually distant dependent due to the limited sensing/communication capabilities of the agents, thus making it difficult or even impossible to satisfy and verify the connectivity assumption for arbitrary initial states.

Motivated by the significance and the practical need for connectivity preservation, connectivity-preserving flocking of mobile networks is rapidly becoming a hot research topic, and various strategies have been developed including both centralized [11,12] and decentralized approaches [6,13–19,21,30,36], which can be divided into three main categories: geometrical constraint technique, spectral graph theory method, and APF method. The geometrical constraint technique first appeared in [10], and was extended to the second-order system in [20]. Through measuring the geometric connectivity robustness of the networked robots, global connectivity could be achieved. For the spectral graph theory method. One branch is to maximize the algebraic connectivity of the graph Laplacian matrix via nonconvex optimization based on subgradient or semidefinite programming (SDP) to guarantee connectivity [11,15]. The other branch is to maintain connectivity via energy functions combined with distributed eigenvalue estimators [22]. The APF method steers the system to converge to the desired flocking configuration while preserving connectivity via superposition of the attractive and repulsive forces, the idea of which is to assign each communication link an appropriate weight characterized as the tension force, which reaches infinity whenever the link tends to break. Other solution techniques include hybrid control laws adopting market-based auctions with gossip algorithms for connectivity preserving link additions and deletions [17].

To the best of our knowledge, many APF based approaches and the spectral graph approaches use unbounded potential fields/enegy functions to force the agents to shrink the communication links whenever they tend to leave the sensing or communication range between each other. The algorithms therein can not guarantee convergence and connectivity maintenance whenever upper bounds on the actuation are imposed. In practical applications, however, unbounded control input is impossible because real mobile agents have only limited actuation capabilities, *e.g.*, the motor cannot generate an

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infinitely large torque to the robots. Although bounded APFs were carefully designed to produce the bounded control inputs in [29,31–33], only agents with first-order kinematics are considered. Furthermore, as the authors pointed out, the development therein is based on the assumption that the probability of simultaneous collisions is negligible, and no other agents stay within the collision region of an agent when it is close to breaking an existing link, which is rather restrictive and hard to be satisfy in real applications. On the other hand, more stringent constraints are required in many existing results on flocking with a dynamic leader that either the velocity of the leader is constant [19,28] or the acceleration of the leader is available to all the followers [8,25,26,30].

1.2 Contributions

To overcome the above limitations, we focus on studying distributed connectivity-preserving flocking algorithm with a dynamic leader for the agent group with double integrator dynamics. The contributions of our work are two fold:

1. In contrast to the unbunded APFs in [17,19], a class of smooth and bounded APFs is carefully designed for flocking behavior by integrating connectivity maintenance, distance stabilization and collision avoidance, simultaneously. Therefore, the proposed local control protocols respect both sensor limitation and actuator saturation and thus are preferable for application in real situations.
2. In the case where none of the followers have access to the acceleration of the dynamic leader, a set of bounded distributed leader-follower flocking control protocols is also carefully devised with the aid of distributed sliding mode control technique, the advantage of which over [8,24–26,28] lies in the accomplishment of the stable flocking motion under more general and practical assumptions without acceleration measurements of the dynamic leader, which might be used to replace certain expensive measurement devices in practical applications.

The rest of the paper is organized as follows: Section II provides background and necessary preliminaries used throughout this paper. Sections III is the main part of the paper which presents the bounded leader-follower flocking control algorithms with connectivity maintenance for second-order multi-agent systems without/with acceleration measurements of the dynamic virtual leader. Nontrivial simulations and experiments are performed in Section IV. Finally, concluding remarks and future directions are given in Section V.

II. PRELIMINARIES

2.1 Background

Some of the main notions in algebraic graph theory which are borrowed from [42] are summarized here. Given N mobile agents, each agent is considered to have limited communication radius R. The communication architecture can be modeled as an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of all agents. $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of communication links among agents. $\mathcal{N}_i$ is the neighbor set of agent i with symmetry property $i \in \mathcal{N}_j$ implies $j \in \mathcal{N}_i$. Agent j is a neighbor of agent i if $(j, i) \in \mathcal{E}$. Proximity-limited communication is modeled by the adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ with the element a_{ij} defined as

$$a_{ij}(t^+) = \begin{cases} 0, & \text{if } ((a_{ij}(t^-) = 0) \wedge (\|x_{ij}(t)\| \geq R - \varepsilon_2)) \\ & \vee((a_{ij}(t^-) > 0) \wedge (\|x_{ij}(t)\| \geq R)) \\ 1, & \text{otherwise} \end{cases} \tag{1}$$

where t^- and t^+ refer to the time instant before and after state transitions of the switching signal and the symbol $\wedge$ and $\vee$ denote the boolean AND and OR operation, respectively. It is worth noting that the $\varepsilon_2 > 0$ introduces a hysteresis region within which the link preserves its membership status (no addition or deletion) and is crucial in stability analysis of the overall system.

Further, the degree matrix of the graph is defined as $D = \mathrm{d}iag\{d_i\}$ with the weighted node degrees $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ as the diagonal elements. The Laplacian matrix of $\mathcal{G}$ is then given by $L = D - A$, which captures the adjacency relationship and the interaction strength between neighboring agents, and has the following properties:

1. L is positive semi-definite for undirected graphs with the eigenvalues of $0 = \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$;
2. $L\mathbf{1_N} = 0$, where $\mathbf{1_N}$ is the column vector of all ones and
3. $\lambda_2(L) > 0$ if and only if the undirected graph $\mathcal{G}$ is connected.

Lemma 1 [23]. Consider a nonnegative matrix H defined as $H = \mathrm{diag}(h_1, h_2, \dots, h_N)$ with at least one positive element, then all eigenvalues of the matrix $L + H$ are positive. Moreover, if $\mathcal{G}_1$ is a graph generated by adding some edges into $\mathcal{G}$, then $\lambda_1(L_1 + H) \geq \lambda_1(L + H) > 0$, where L_1 is the graph *Laplacian* of $\mathcal{G}_1$.

2.2 Problem formulation

Consider a group of N agents with second-order dynamics moving in the plane, which is described by

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= u_i \end{aligned} \quad i = 1, 2, \dots, N \tag{2}$$

where x_i is the position vector of agent i, v_i is the velocity vector of agent i, u_i is the control input (acceleration) acting on agent i. Denote $x = (x_1^T, x_2^T, \dots, x_N^T)^T \in \mathbb{R}^{2N}$ and $v = (v_1^T, v_2^T, \dots, v_N^T)^T \in \mathbb{R}^{2N}$ to be the stack position and velocity vectors of the entire system. Let $\varepsilon \in (0, R]$ be a small hysteresis constant. Initial links are generated by $\mathcal{E}(0) = \{(i, j) \mid \|x_i(0) - x_j(0)\| < R - \varepsilon_0, i, j \in \mathcal{V}\}$, where $0 < \varepsilon_0 \leq \varepsilon_2$.

Furthermore, a dynamic leader is introduced to guide a group of agents and treated as an ordinary agent by each agent called a follower. Each follower can sense the information of the dynamic leader if only if the distance between them is less than the communication radius. The motion of the dynamic leader is described by $\dot{x}_l = v_l$, where $x_l \in \mathbb{R}^2$ and $v_l \in \mathbb{R}^2$ denote the position and velocity vectors of the dynamic leader. Here, without loss of generality, it is assumed that $\|\dot{v}_l\|_1 < f$, where f is a positive constant.

The control objective here is to derive a set of bounded distributed controllers using only local information to steer the followers to achieve velocity consensus and collision avoidance with the dynamic leader with time-varying velocity, while guaranteeing the connectivity of the underlying communication graph as the system evolves, provided the given graph is initially connected.

III. MAIN RESULTS

3.1 Leader-follower flocking algorithm with connectivity preservation

It is worth noting that the algorithms in [8,25,26,30] are feasible only when each follower can access the accurate acceleration information of the dynamic leader, *i.e.*, $\dot{v}_l$. However, since not all the agents (robots, air vehicles, manipulators, *etc.*) in practice are equipped with acceleration sensors, acceleration measurements are more difficult to obtain than position and velocity measurements. Moreover, an algorithm in the absence of acceleration measurements has the advantage of decreasing equipment cost and network traffic. Therefore, we are motivated to design distributed connectivity-preserving leader-follower flocking algorithms without using acceleration measurements.

To achieve the desired flocking motion, the explicit bounded flocking control protocol for each follower i is devised as follows:

$$\begin{aligned} u_i = &-\sum_{\substack{j\in\mathcal{N}_i\\ j\neq l}} \nabla_{x_i} V_{ij}(\|x_{ij}\|) - h_i \nabla_{x_i} V_{il}(\|x_{il}\|) \\ &- \alpha \sum_{j\in\mathcal{N}_i} a_{ij}\left\{\mathrm{sgn}\left\{\sum_{\substack{k\in\mathcal{N}_i\\ k\neq l}} a_{ik}(v_i - v_k) + h_i(v_i - v_l)\right\}\right\} \\ &+ \alpha \sum_{j\in\mathcal{N}_i} a_{ij}\left\{\mathrm{sgn}\left\{\sum_{\substack{k\in\mathcal{N}_j\\ k\neq l}} a_{jk}(v_j - v_k) + h_j(v_j - v_l)\right\}\right\} \end{aligned} \tag{3}$$

where $h_i(t) = \begin{cases} 0, & i \in \mathcal{N}_l(t) \\ 1, & \text{otherwise} \end{cases}$ $sgn(\cdot)$ is the signum function, $\alpha > 0$ is the control gain, V_{ij}, $\forall j \in \mathcal{N}_i$ is the bounded interactive APF between agents i and j which is to be designed.

Remark 1. Note that, inspired by [27,35], the sliding mode control approach is adopted here to serve as a distributed estimator for all the followers without acceleration measurements to the dynamic leader. The asymptotic stable flocking behavior of the entire system can be guaranteed by properly adjusting the control gain α, the determination of which will be detailed later. The distinguished features of the proposed control protocol (3) lie in the removal of the mild connectivity requirement in [27,35] and the bounded control force.

Then, define the positive semi-definite function energy function as

$$\begin{aligned} \psi(x, v, x_l, v_l) = &\frac{1}{2}\sum_{i=1}^{N}(v_i - v_l)^T(v_i - v_l) \\ &+ \frac{1}{2}\sum_{i=1}^{N} U_i(x, x_l) \end{aligned} \tag{4}$$

where

$$U_i(x, x_l) = \sum_{\substack{j\in\mathcal{N}_i\\ j\neq l}} V_{ij}(\|x_{ij}\|) + 2h_i V_{il}(\|x_{il}\|)$$

Note that U contains all the existing follower-follower potentials and the leader-follower tracking potentials, which have the physical meaning of characterizing all the interactive potential energy of the entire system.

Further, define $\Psi_{\max}$ which satisfies

$$\begin{aligned} \psi_{\max} = &\frac{1}{2}\sum_{i=1}^{N}(v_i(0) - v_l(0))^T(v_i(0) - v_l(0)) \\ &+ \frac{N(N+1)}{2}V_{\max} \end{aligned} \tag{5}$$

Remark 2. Note that the first term and the second term in (5) indicate the initial kinetic energy and the possible maximum potential energy of the system, respectively. The combination of both gives the total maximum mechanical energy and the overall leader-follower multi-agent system in the context of the complete undirected graph, which is vital for designing the bounded **APF** V_{ij}.

In order to enable the overall system to achieve desired stable flocking motion using only bounded control inputs, $V_{ij}(\|x_{ij}\|)$ should be well designed to be a bounded and nonnegative potential of the distance of $\|x_{ij}\| = \|x_i - x_j\|$ while integrating requirements of connectivity maintenance and collision avoidance, such that:

1. $V_{ij}(\|x_{ij}\|)$ is continuously differentiable for $\|x_{ij}\| \in (0, R)$;
2. $V_{ij}(\|x_{ij}\|)$ is monotonically decreasing for $\|x_{ij}\| \in (0, d)$ and monotonically increasing for $\|x_{ij}\| \in (d, R)$, where $\varepsilon_1 < d < R - \varepsilon_2$ and
3. $V_{ij}(0) = \Psi_{\max}$ and $V_{ij}(R) = \Psi_{\max}$.

$$\begin{aligned} \psi_{\max} \triangleq &\frac{1}{2}\sum_{i=1}^{N}(v_i(0) - v_l(0))^T(v_i(0) - v_l(0)) \\ &+ \frac{N(N+1)V_{\max}}{2} \end{aligned} \tag{6}$$

where $V_{\max} = \max\{V(\varepsilon_1), \quad V(R - \varepsilon_2)\}$ and $\varepsilon_1 = \min_{i,j\in\mathcal{E}(0)}\{\|x_{ij}(0)\|_2\}$.

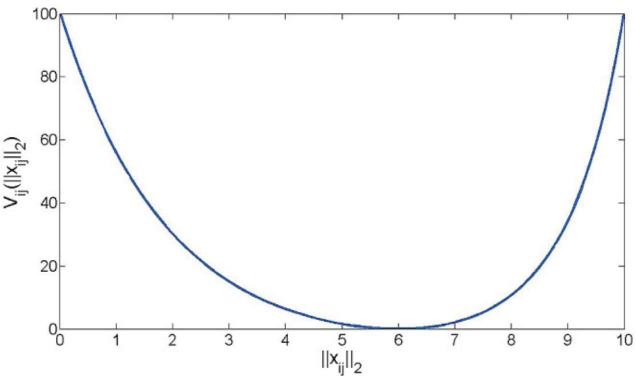

Fig. 1. An example of V_{ij} with $R = 10$, $d = 6$ and $\Psi_{\max} = 100$.

Condition (i) aims at producing smooth controller for each agent; Condition (ii) illustrates that the potential V_{ij} provides an attractive or repulsive force between agents i and j when their distance tends to R or zero. Clearly, V_{ij} reaches its minimum at $\|x_{ij}\| = d$; Condition (iii) states that the potential will be sufficiently large when the distance between them reaches R or zero, which can guarantee both connectivity maintenance and collision avoidance. One candidate example is given below

$$V(\|x_{ij}\|) = \frac{(\|x_{ij}\| - d)^2 (R - \|x_{ij}\|)}{\|x_{ij}\| + \dfrac{d^2 (R - \|x_{ij}\|)}{\psi_{\max}}} + \frac{\|x_{ij}\| (\|x_{ij}\| - d)^2}{(R - \|x_{ij}\|) + \dfrac{\|x_{ij}\| (R-d)^2}{\psi_{\max}}} \tag{7}$$

Note that in [14,16,17], two specific potential functions are introduced and tend to infinity when the relative distance between two agents i and j tends to R, which may not be practical because of the infinitely large (unbounded) control effort.

Remark 3. Note that determination of $\varepsilon_1 = \min_{i,j\in\mathcal{E}(0)} \{\|x_{ij}(0)\|_2\}$ requires the global information of all the initial relative distances between neighboring agents, however, the gossip algorithms combined with auction-based consensus decision mechanisms in [37] can be utilized to pre-determine ε_1 in a totally distributed fashion. The details of the algorithms are out of the scope of current paper and hence omitted here to save space.

Remark 4. From Fig. 1 and (7), one can obtain that both V_{ij} and its gradient-based term are bounded. α is the bounded control gain. Furthermore, since the signum function $sgn(\cdot)$ is also bounded in $[-1,1]$, the control input (3) for each agent is bounded by considering both connectivity maintenance and collision avoidance, which is different from the existing second-order leader-follower flocking algorithms in [8,25,26,30].

3.2 Stability analysis

Assume that $\mathcal{G}(t)$ switches at time t_k, $k = 1, 2, \ldots$ and $t_0 = 0$, recall that the link hysteresis mechanism introduces a dwell time $\tau > 0$ between consecutive switches in the network topology. Then the main result is then stated in the following theorem.

Theorem 1. Consider a second-order multi-agent system that consists of N followers and a dynamic leader moving with dynamics (2), and with the followers steered by protocol (3). Suppose that the initial communication graph $\mathcal{G}(0)$ is connected, the initial energy $\psi(0)$ is finite, $t_k - t_{k-1} > \tau > 0$ for all switching times t_k and the control gain $\alpha > f/\|(L(0) + H(0))\|_1$. Then $\mathcal{G}(t)$ will remain to be connected for all $t \geq 0$, all the agents asymptotically attain the same velocity with the dynamic leader with guaranteed collision avoidance.

Proof. Denote the position difference and the velocity difference between follower i and the dynamic virtual leader l as $\tilde{x}_i = x_i - x_l$ and $\tilde{v}_i = v_i - v_l$, respectively. Then we have

$$\begin{cases} \dot{\tilde{x}}_i = \tilde{v}_i \\ \dot{\tilde{v}}_i = -\sum\limits_{\substack{j\in\mathcal{N}_i \\ j\neq l}} \nabla_{\tilde{x}_i} \psi(\|\tilde{x}_i - \tilde{x}_j\|) - h_i \nabla_{\tilde{x}_i} V_{il}(\|\tilde{x}_i\|) - \dot{v}_l \\ \qquad -\alpha \sum\limits_{j\in\mathcal{N}_i} a_{ij} \left\{ \operatorname{sgn}\left\{ \sum\limits_{\substack{k\in\mathcal{N}_i \\ k\neq l}} a_{ik}(\tilde{v}_i - \tilde{v}_k) + h_i \tilde{v}_i \right\}\right\} \\ \qquad +\alpha \sum\limits_{j\in\mathcal{N}_i} a_{ij} \left\{ \operatorname{sgn}\left\{ \sum\limits_{\substack{k\in\mathcal{N}_j \\ k\neq l}} a_{jk}(\tilde{v}_j - \tilde{v}_k) + h_j \tilde{v}_j \right\}\right\} \end{cases} \tag{8}$$

Moreover, (4) can be rewritten as

$$\psi(\tilde{x}, \tilde{v}) = \frac{1}{2}\sum_{i=1}^{N} U_i(\tilde{x}, \tilde{v}) + \frac{1}{2}\sum_{i=1}^{N} \tilde{v}_i^T \tilde{v}_i \tag{9}$$

where $\tilde{x} = [\tilde{x}_1^T, \ldots, \tilde{x}_N^T]^T$, $\tilde{v} = [\tilde{v}_1^T, \ldots, \tilde{v}_N^T]^T$ and

$$U_i(\tilde{x}) = \sum_{\substack{j\in\mathcal{N}_i \\ j\neq l}} V(\|\tilde{x}_i - \tilde{x}_j\|) + 2h_i V_{il}(\|\tilde{x}_i\|)$$

During $[0, t_1)$, since $\psi(0)$ is finite, take the time derivative of ψ, we have

$$\begin{aligned} \dot{\psi} &= -\sum_{i=1}^{N} \tilde{v}_i^T \left[\alpha \sum_{j\in N_i} a_{ij} \left\{ \operatorname{sgn}\left\{ \sum_{\substack{k\in\mathcal{N}_i \\ k\neq l}} a_{ik}(\tilde{v}_i - \tilde{v}_k) + h_i \tilde{v}_i \right\}\right\}\right] \\ &\quad + \sum_{i=1}^{N} \tilde{v}_i^T \left[\alpha \sum_{j\in N_i} a_{ij} \left\{ \operatorname{sgn}\left\{ \sum_{\substack{k\in\mathcal{N}_j \\ k\neq l}} a_{ik}(\tilde{v}_j - \tilde{v}_k) + h_j \tilde{v}_j \right\}\right\}\right] \\ &\quad - \sum_{i=1}^{N} \tilde{v}_i^T \dot{v}_l \\ &= -\alpha \tilde{v}^T (L(0) + H(0)) \operatorname{sgn}[(L(0) + H(0))\tilde{v}] - \sum_{i=1}^{N} \tilde{v}_i^T \dot{v}_l \\ &\leq (f - \alpha\|L(0) + H(0)\|_1)\|\tilde{v}\|_1 \leq 0, \quad \forall t \in [t_0, t_1) \end{aligned} \tag{10}$$

which implies that $\Psi(t) \le \Psi(0) < \Psi_{\max}$, $\forall t \in [0, t_1)$. Therefore, no distance of existing edges will tend to R and no distance of neighboring agents will tend to 0 for $t \in [0, t_1)$. Otherwise from (7), $V_{ij}(0) = \Psi_{\max} > \Psi_{\max}$, $V_{ij}(R) = \Psi_{\max} > \Psi_{\max}$, which results in contradiction. Therefore, no collisions will occur and no existing edges will be lost before time t_1. Therefore, new edges must be added to the underlying network at the switching time t_1. Note that the hysteresis ensures that if a finite number of edges are added to $\mathcal{G}(t)$, then the associated potential remains finite, thus $\psi(t_1)$ is finite.

From Lemma 1 and $\alpha > f/\|L(0)+H(0)\|_1$, it follows that $\alpha > f/\|L(t_{k-1})+H(t_{k-1})\|_1$. Similar to the above analysis, on each $[t_{k-1}, t_k)$, taking the time derivative of ψ, we have

$$\begin{aligned}\dot{\psi} &= -\alpha\tilde{v}^T\left((L+H)(t_{k-1})\right)\mathrm{sgn}\left[\left((L+H)(t_{k-1})\right)\tilde{v}\right] \\ &\quad -\sum_{i=1}^{N}\tilde{v}_i^T\dot{v}_l \\ &\le \left(f-\alpha\|(L+H)(t_{k-1})\|_1\right)\|\tilde{v}\|_1 \le 0\end{aligned} \tag{11}$$

which implies that

$$\begin{aligned}&\psi(t) \le \psi(t_{k-1}) < \psi_{\max} \\ &\forall t \in [t_{k-1}, t_k), \quad k = 2, 3, \ldots\end{aligned} \tag{12}$$

Then, following the same analysis, since $\psi(t)$ is bounded for all $t \ge 0$, no distance of existing edges will tend to R or zero as the system evolves, which indicates that no edges will be lost and no collisions among neighboring agents will occur for all $t \in [t_{k-1}, t_k)$, and $\psi(t_k)$ is finite. Because is connected and no edges in $\mathcal{E}(0)$ were lost, will remain connected for all $t \ge 0$.

Suppose there are N_k new links being added to $\mathcal{G}(t)$ at t_k, it is known that $0 < N_k \le N(N+1)/2 - N = N(N-1)/2 \triangleq \bar{N}$, from (4) and (11), we have

$$\psi(t_k) \le \psi(0) + (N_1 + \cdots + N_k)V(\|R-\varepsilon_2\|) < \psi_{\max} \tag{13}$$

Due to the fact that there are at most $\bar{N}$ new links that can be added for any initially connected network, we have $N_k < \bar{N}$ and $\Psi(t) < \Psi_{\max}$, $\forall t \ge 0$. Thus, the number of switching times k is finite, which implies $\mathcal{G}(t)$ finally becomes fixed. Therefore, the following discussions are restricted on the time interval $[t_k, +\infty)$, note that all the edges are no longer than $V^{-1}(\Psi_{\max})$ due to the monotonically increasing feature of V in $[d, R)$ defined in (7). Further define the level set

$$\Omega = \{\bar{\tilde{x}} \in D, \tilde{v} \in R^{2N} \mid \psi(\tilde{x}, \tilde{v}) \le \psi_{\max}\}$$

which is a positively invariant set, where

$$\begin{aligned}D = \{&\bar{\tilde{x}} \in R^{N^2} \mid \|\tilde{x}_i - \tilde{x}_j\|_2 \in [0, V^{-1}(\psi_{\max})], \\ &\forall (i, j) \in \mathcal{E}(t)\}\end{aligned}$$

where $\bar{\tilde{x}} = [\tilde{x}_{11}^T, \cdots, \tilde{x}_{1N}^T, \cdots, \tilde{x}_{N1}^T, \cdots, \tilde{x}_{NN}^T]^T$.

As $\mathcal{G}(t)$ is connected for all $t \ge 0$, it follows that $\|\tilde{x}_i - \tilde{x}_j\|_2 \le (N-1)R$ for all agents i and j. Due to the fact that $\Psi(t) < \Psi_{\max}$, we have $\|\tilde{v}_i\|_2 < \sqrt{2\psi_{\max}}$, $\forall i$. Hence, Ω is closed and bounded, therefore compact. Note that system (2) with control input (3) is an autonomous system on the concerned time interval $[t_k, \infty)$. Therefore, it follows from the LaSalle invariance principle for nonsmooth systems [41] that the trajectories will converge to the largest invariant set inside the region

$$S = \{\bar{\tilde{x}} \in D, \tilde{v} \in R^N \mid \dot{\psi} = 0\} \tag{14}$$

From (11), $\dot{\psi} = 0 \Leftrightarrow \|\tilde{v}\|_1 = 0 \Leftrightarrow v_1 = \cdots = v_N = v_l$, which implies the velocities of all the followers will converge to that of the virtual leader asymptotically. Since $v_1 = \ldots = v_N = v_l$, it can be easily deduced that $\frac{d\|x_{ij}\|_2^2}{dt} = 2x_{ij}^T(v_i - v_j) = 0, \forall (i, j) \in \mathcal{E}(t)$, which indicates that the interagent distance is stabilized in steady state.

IV. SIMULATION AND EXPERIMENT

4.1 Simulation

In this subsection, comparative numerical simulations are performed to compare our leader-follower connectivity maintenance flocking algorithm with the flocking algorithms proposed by [8,25,26] in the same initial state. For simplicity and without loss of generality, the simulations are performed with five agents moving in the plane, the initial positions of all agents are set within the box of [0, 10] m × [0, 10] m such that the communication network is initially connected, and the initial velocities of all agents are chosen randomly in the box of [−5, 5] m/*s* × [−5, 5] *m*/*s*. Therefore, the velocity bound for all the agents is $v_{\max} = 5$ m/s.

It is assumed that the initial time $t_0 = 0$s and the simulations are run for a time period of 60 s, where the dynamic leader follows a circular trajectory tracking and a sine shape trajectory tracking, respectively. The specific dynamics of the leader are given as below:

$$u_l = -3\cos(t)[1, 0]^T - 3\sin(t)[0, 1]^T \tag{15}$$

while the remaining followers apply control protocols (3). Furthermore, the potential function V is defined in (7) with the communication radius fixed at $R = 1$ m, the desired distance $d = 0.5$ m and $\varepsilon_0 = \varepsilon_2 = 0.1$, $\varepsilon_1 = 0.3$. The control gain α is set to 10. Some simple calculations give that $V_{\max} = V(R - \varepsilon_2)$. Then we have

$$\begin{aligned}\psi_{\max} &\le \frac{1}{2}\sum_{i=1}^{N}(v_i(0) - v_l(0))^T(v_i(0) - v_l(0)) \\ &\quad + \frac{N(N+1)}{2}V(R-\varepsilon_2) \\ &\le 2Nv_{\max}^T v_{\max} + \frac{N(N+1)}{2}V(R-\varepsilon_2) \\ &= 2Nv_{\max}^T v_{\max} + \frac{N(N+1)}{2}\frac{(R-\varepsilon_2-d)^2\varepsilon_2}{(R-\varepsilon_2)+\frac{d^2+\varepsilon_2}{\psi_{\max}}} \\ &\quad + \frac{N(N+1)}{2}\frac{(R-\varepsilon_2)(R-\varepsilon_2-d)^2}{\varepsilon_2+\frac{(R-\varepsilon_2)(R-d)^2}{\psi_{\max}}}\end{aligned} \tag{16}$$

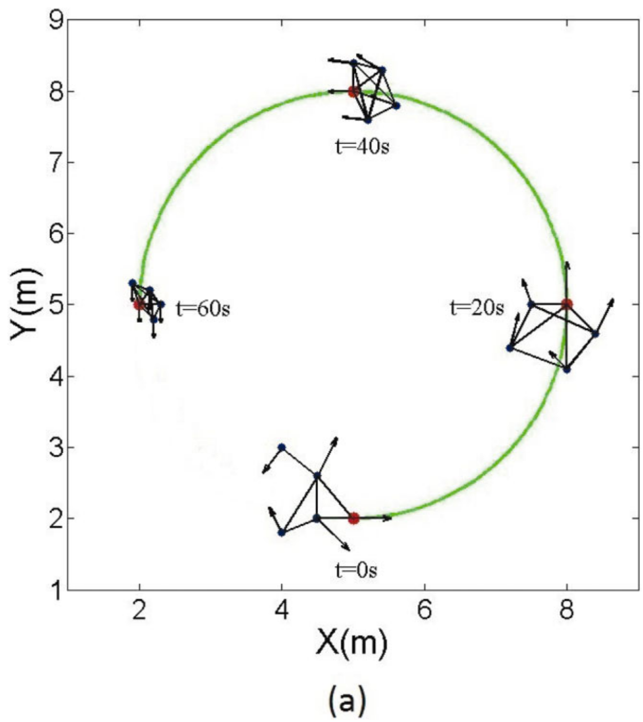

Fig. 2. Flocking with a dynamic leader following a circular curve.

Therefore, $\Psi_{\max}$ can be evaluated by using an estimate of the bound of the initial velocities of all the agents, and one gets $\Psi_{\max} \le 50.3$. Then we further have the explicit form of the bounded potential function

$$V_{ij}(\|x_{ij}\|) = \frac{(\|x_{ij}\| - 0.5)^2(1-\|x_{ij}\|)}{\|x_{ij}\| + \dfrac{(1-\|x_{ij}\|)}{240}} + \frac{\|x_{ij}\|(\|x_{ij}\| - 0.5)^2}{(1-\|x_{ij}\|) + \dfrac{\|x_{ij}\|}{240}} \tag{17}$$

Then substituting (17) into (3) yields the final explicit controllers for all the followers.

Fig. 2 depicts the consecutive snapshots of the agent configurations throughout the simulation to illustrate the process of flocking with a dynamic leader following the circular curve under the control law (3), respectively. All the followers are labeled with blue dots and the dynamic leader is labeled with large solid red dot whose dynamics is independent of all the other agents. The black solid lines denote the communication links and the black solid lines with arrows represent the velocities of all agents. The thick green line denotes the desired trajectory followed by the dynamic leader, which is a circle centered at $(5, 5)^T$ m with radius $R_c = 3$ m. The initial position of the dynamic leader is set as $[5, 2]^T$ m, which is located at the bottom of the circle.

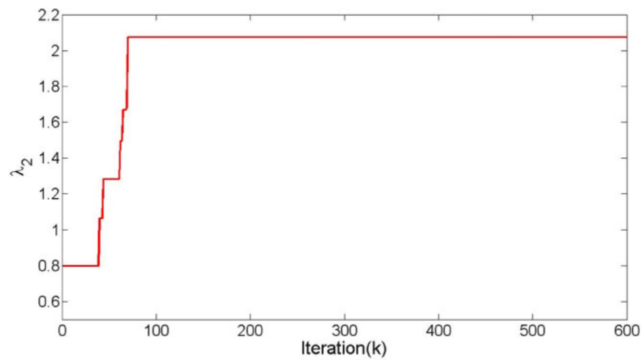

Fig. 3. Algebraic connectivity under (3) following a circular curve.

Fig. 3 shows the evolving curve of the algebraic connectivity of the underlying communication network. It can be seen that for either case, as the dynamic leader traverses a trajectory through the workspace the remaining followers attempt to maintain and increase network connectivity while avoiding interagent collisions. Fig. 4 and Fig. 5 show the comparative results of the velocity curves of all agents for circular trajectory tracking under control law (3) with the control laws in [8,25,26], respectively. The curves of the control inputs of all followers are ploted in Fig. 6. It can be clearly seen that for the same initial states, the algorithms proposed in [8,25,26] can not guarantee velocity consensus of all followers with the dynamic leader without acceleration measurements, which results in failure in tracking the dynamic leader as well as the stable flocking motion. On the contrary, with our proposed control protocol (3), the velocities asymptotically obtain the same as the dynamic leader without collisions in either case. The stable group flocking behavior is generated eventually with only bounded control inputs, which verifies the effectiveness and demonstrates the advantage of our presented control algorithm.

4.2 Experiment

Finally, the experimental verification for leader-follower flocking with real mobile robots is conducted in a real indoor environment, we use five wheeled mobile robots which consist of 2 Pioneer 3-AT robots and 3 Amigobots. The initial positions of the robots are chosen randomly from the boxes of [0, 9] m × [0, 9] m such that the neighboring graph is initially connected. The initial velocities of the robots are randomly chosen in the boxes of [−2, 2] m/s × [−2, 2] m/s. Each robot can obtain the information needed via its wireless communication equipment and the sensing equipment, and the control period is $\Delta(t) = 0.05$s. The following parameters remain fixed through the experiments: $R = 4$, $d = 2$, $\delta = 0.8$, $\varepsilon_0 = \varepsilon_2 = 0.3$, $\varepsilon_1 = 0.8$, $\alpha = 1.5$. Following the same analysis, we can get $\Psi_{\max} = 408.35$ and the final explicit form of the bounded APF as below

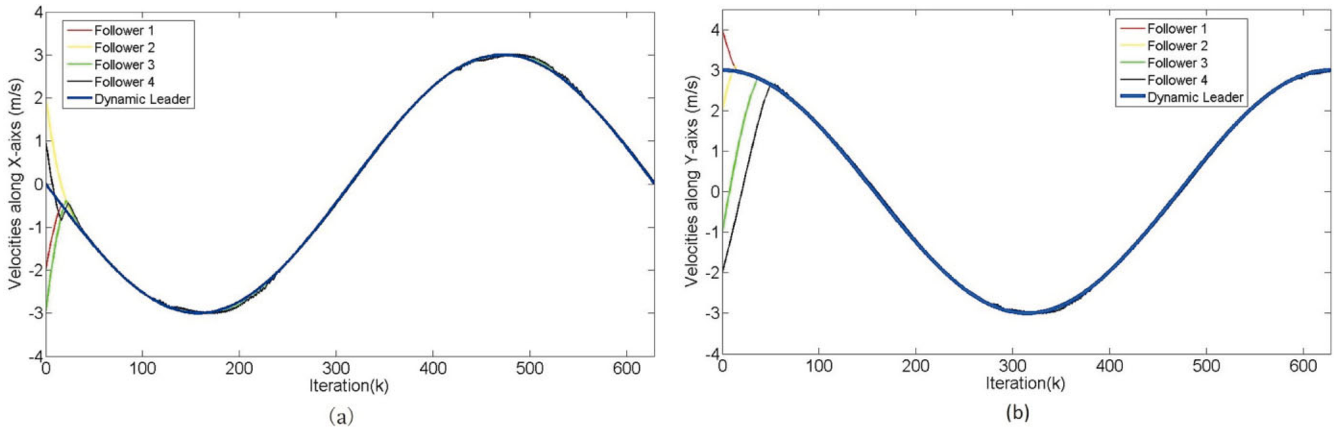

Fig. 4. Velocities of all agents for circular tracking using (3).

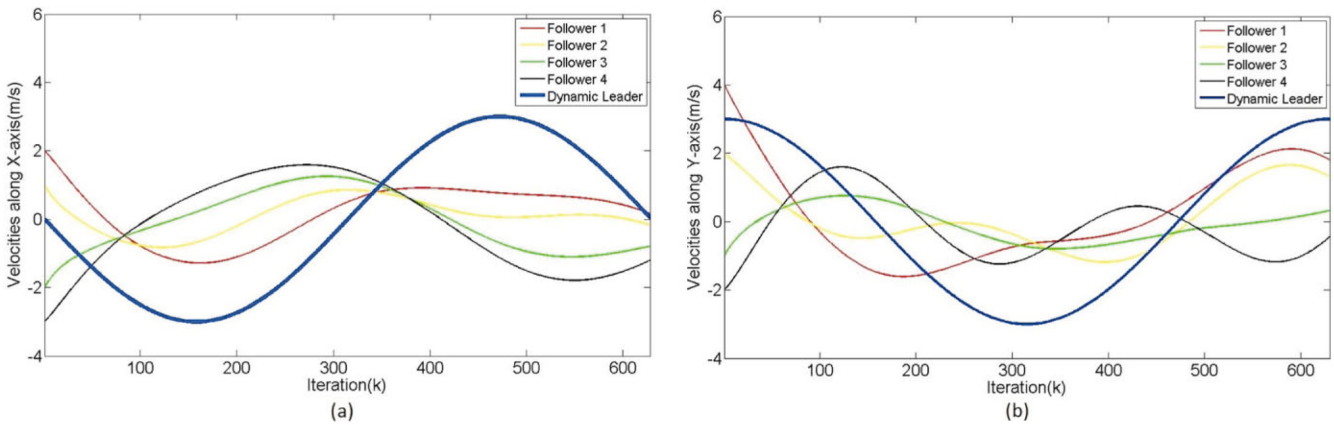

Fig. 5. Velocities of all agents for circular tracking using [28].

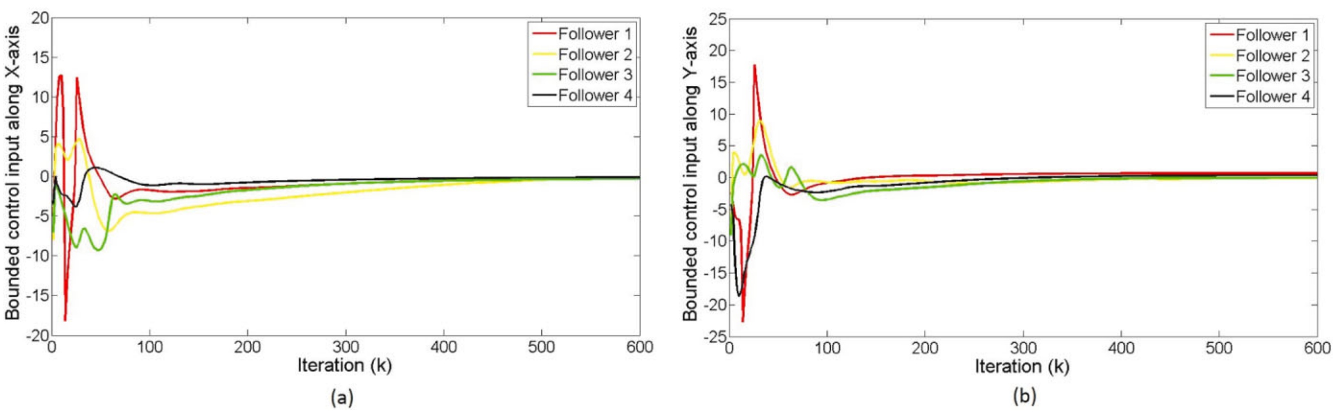

Fig. 6. Velocities of all agents for circular tracking without using (3).

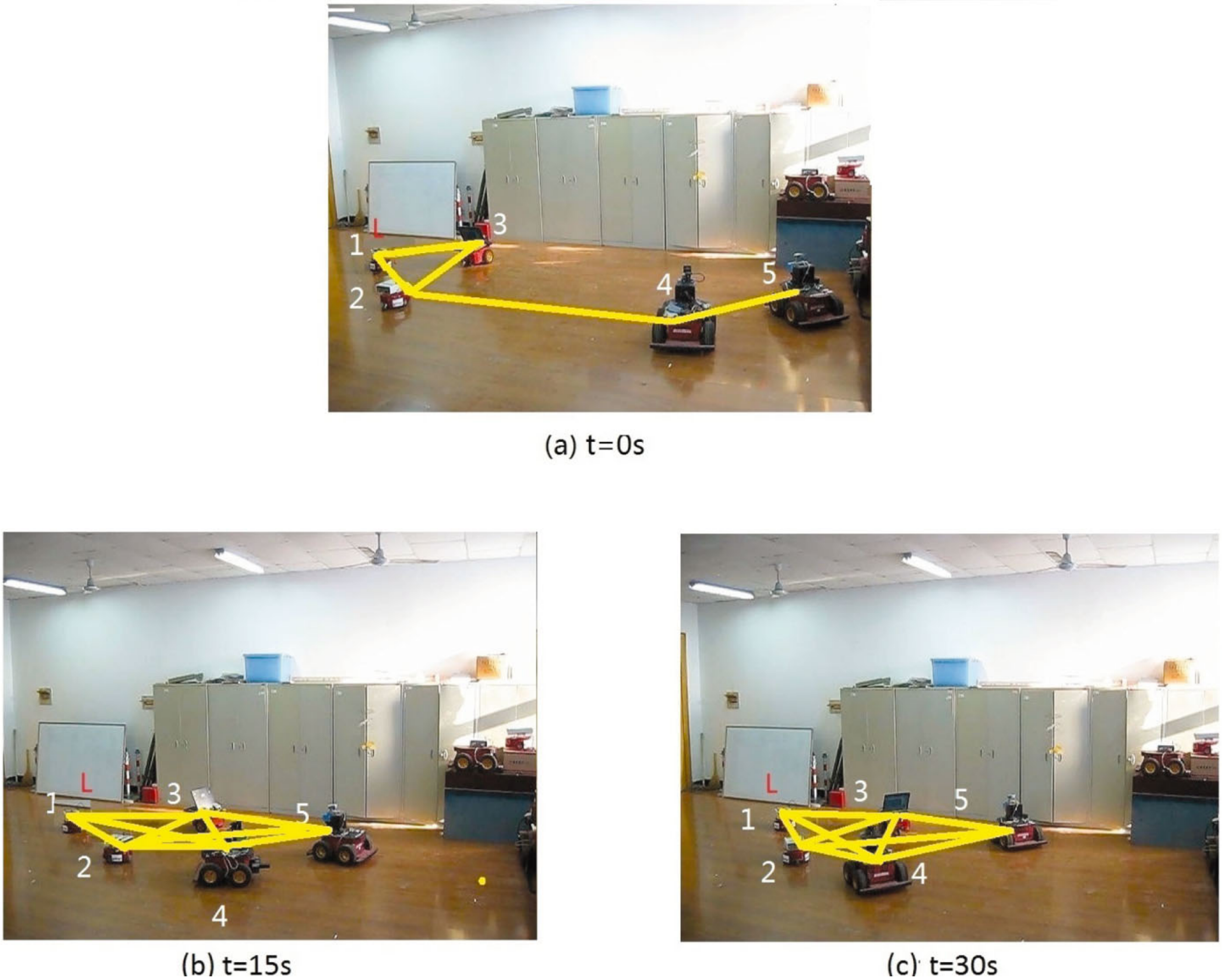

Fig. 7. Flocking experiment of 5 wheeled mobile robots in indoor environment with a dynamic leader.

$$V_{ij}(\|x_{ij}\|) = \frac{(\|x_{ij}\| - 2)^2(4 - \|x_{ij}\|)}{\|x_{ij}\| + \frac{(4 - \|x_{ij}\|)}{105}} + \frac{\|x_{ij}\|(\|x\|_{ij} - 2)^2}{(4 - \|x_{ij}\|) + \frac{\|x_{ij}\|}{105}} \tag{18}$$

Fig. 7 illustrates three consecutive snapshots during the whole process of leader-follower flocking within a time frame of 30 seconds. Without loss of generality, the initially connected topology consists of 4 followers and 1 dynamic leader which is shown in Fig. 7a. The dynamics of the leader are set as $v_l(0) = [1, 0.5]^T$ m/s and $u_l(t) = [0.1,\ 0.05]^T$ m^2/s. The respective graphical elaborations better illustrate the robots connections, in particular, the yellow lines show the communication links among the neighboring robots, the numbers beside every robot denotes different IDs of the robots. In addition, the dynamic leader is denoted by a capital "L". Fig. 7b demonstrates that the initially sparsely connected agents are moving towards the dynamic leader and become a cohesively connected group without collisions due to the attraction and repulsion forces generated by the interactive potentials. Fig. 7c depicts the final state which shows that all the followers move coherently with the dynamic leader. The velocity curves of all the agents are shown in Fig. 8. The relative distances between each follower and the leader are depicted in Fig. 9, where followers 1−4 refer to robots 2−5 respectively. Fig. 10 plots the evolving curve of algebraic connectivity of the underlying communication network. The magnitudes of the control inputs of all the followers are plotted in Fig. 11, which are bounded throughout the process of system

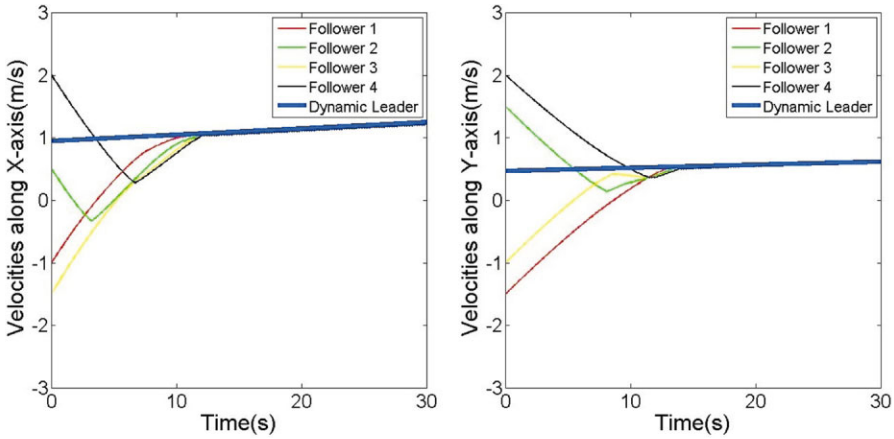

Fig. 8. Velocity curves of all the robots.

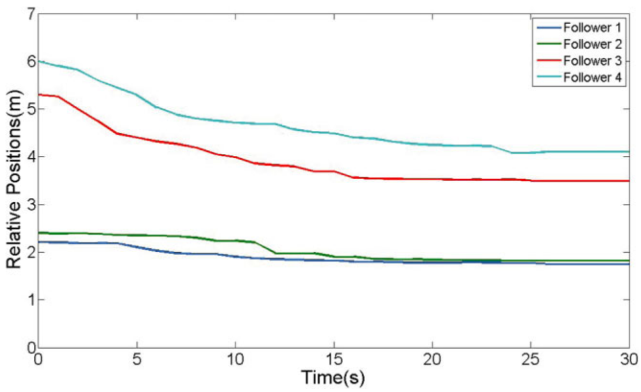

Fig. 9. Relative distances between all followers and the leader.

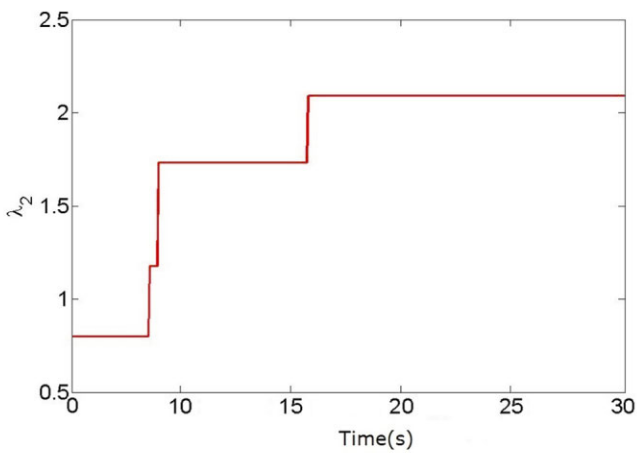

Fig. 10. Algebraic connectivity of the underlying communication network.

evolution. It can be observed that the velocities of all the followers obtain the same as that of the dynamic leader with guaranteed connectivity maintenance, thus the stable desired flocking behavior is successfully achieved eventually using bounded control inputs, which is consistent with the theoretical results.

V. CONCLUSION AND FUTURE WORK

Distributed leader-follower flocking algorithms with connectivity preservation for second-order multi-agent systems using bounded control inputs were presented, and only partial measurements of the states of the dynamic leader are available to the followers. First, a novel class of smooth and bounded APFs was introduced to generate the attractive and repulsive effects of the controller to navigate the followers, which could guarantee connectivity maintenance, collision avoidance and distance stabilization, simultaneously. Moreover, in the absence of acceleration measurements of the dynamic leader to each follower, a set of provably stable leader-follower local flocking control protocols was carefully designed for all the followers to track the dynamic leader with a time-varying velocity through a combination of the variable structure control technique with the potential-based gradient decent methodology accounting for unavailability of the acceleration measurements. On condition that the underlying communication network is initially connected, sufficient conditions related to the control gain were derived for all the followers to asymptotically achieve velocity consensus and collision avoidance with the dynamic leader with guaranteed global connectivity maintenance and distance stabilization, implying the generation of the desired stable flocking behavior eventually. Finally, the effectiveness of the proposed controllers were verified by extensive illustrative simulations and experiments which were shown to be well consistent with the theoretical results.

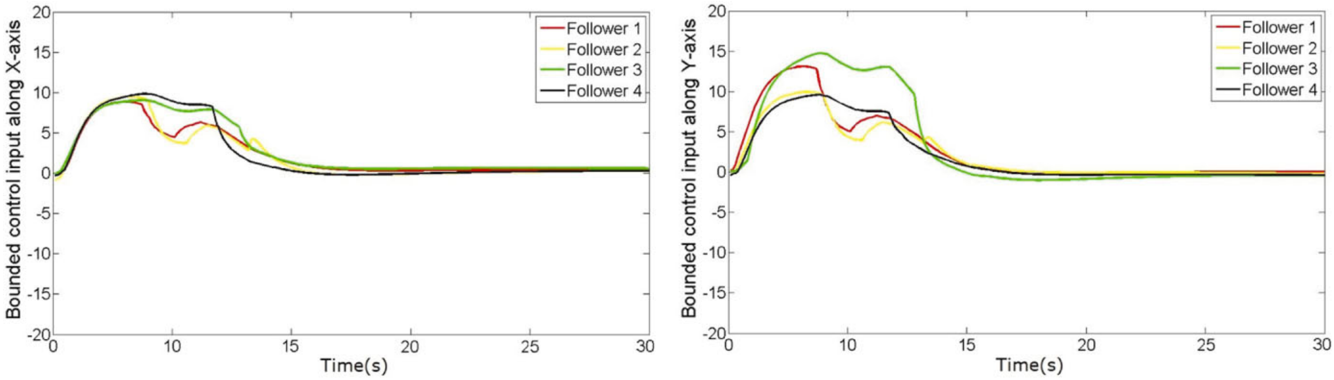

Fig. 11. The magnitudes of the bounded control inputs of all followers.

While the current work complements some existing results in the literature, there are some issues that need to be addressed in the future. First, it will be interesting and challenging to extend the proposed bounded APFs to more complex environmental settings with multiple static/moving obstacles. Second, it will be interesting also to derive convergence conditions under which the flocking behavior can still be achieved in the presence of time delay and random noise, which play an important role in real-world applications. In addition, bounded flocking algorithms for nonlinear multi-agent dynamic systems on directed networks are promising directions deserving further investigations.

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