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Research Article

Dynamic Quantized Predictive Control for Systems with Time-Varying Delay and Packet Loss in the Forward Channel

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Stability and design of a dynamic quantized predictive control system with time-varying delay and packet loss are studied. For the system with time-varying delay and packet loss in the forward channel, a dynamic quantizer that can minimize the quantized output error is designed and a networked quantized predictive control (NQPC) strategy is proposed to compensate for the delay and packet loss. Stability of the NQPC system is then analyzed and a sufficient stability condition is derived and presented in the form of matrix inequality. Finally, both simulation and experimental results are given to demonstrate the effectiveness of the proposed approach.

1. Introduction

In past decades, networked control systems (NCSs) have been widely studied, whose data are transmitted by networks with finite capacity. Different from traditional control systems, there exist many inevitable problems such as network induced delays and packet losses in networked control systems. To overcome such difficulties, some effective methods have been proposed, among which a representative one is the networked predictive control (NPC) method [1–5].

For example, predictive control system design with time-varying delay in the feedback channel was considered in [1], where sufficient conditions for stability of the closed-loop NCS were given. Meanwhile, NPC systems with delays both in forward and feedback channel were investigated in [2, 3]. The implementation of NPC scheme was addressed in [4], where both simulation and practical implementation were carried out. In [5] an event-driven predictive controller was designed and a practical example was presented to confirm the effectiveness of the NPC method.

For control systems that use networks for communication, data quantization is an important problem which should be taken into consideration. Strictly speaking, all networked control systems are quantized control systems, because data quantization is inevitable before transmission.

There are generally two representative quantizers for quantization: the static quantizer and the dynamic quantizer. In [6–8], a static logarithmic quantizer was studied, where stabilization of discrete-time systems was investigated. In [9, 10], stabilization of systems using finite data rates was analyzed, and it was proved that the finite horizon coder is actually a quantizer. In [11, 12], analysis of systems with a quantized feedback was considered by investigating quantizer complexity versus system performance. In [13, 14], the coarsest logarithmic quantizer design and stabilization of quantized system with packet loss were analyzed. Compared with the static quantizer, dynamic quantizer has been an important topic in last several years. In [15, 16], a novel dynamic quantizer with a scaling factor was proposed, and asymptotic stability was studied using the "zooming" approach. In [17], an optimal dynamic quantizer was investigated, which can minimize the output error between the quantized system and the unquantized system.

For the studies mentioned above, quantization has not been considered for NPC systems that is able to compensate time-varying delay and packet loss. Meanwhile, active

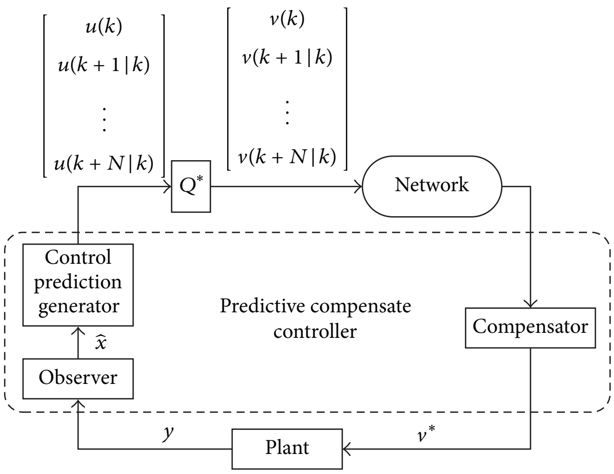

Figure 1: Dynamic quantized predictive control system.

compensation of networked delay and packet loss has not been studied for quantized systems before. For these reasons, in this paper, a synthesis method of NQPC system is given. Both time-varying delay and packet loss are compensated using the predictive control method. An improved dynamic quantizer that can minimize the quantized error in the input-output relation is designed for the NQPC system. The closed-loop system is then lifted to a switched system [18–21] and a sufficient condition for stability is given.

The whole paper is organized as follows. Section 2 describes the NQPC system with time-varying delay and packet loss. Section 3 studies the predictive compensation strategy. Section 4 designs a dynamic quantizer that can minimize the maximum output error of the system. Section 5 analyzes stability of the NQPC system. Section 6 gives both simulation and experimental results to indicate that our control strategy is effective and Section 7 concludes the paper.

2. NQPC System Description

The networked quantized predictive control (NQPC) system with time-varying delay in the forward channel as shown in Figure 1 is studied in this paper. The key idea of NQPC is that all the possible future control inputs are quantized and packed into a single packet before being transmitted through the network. Then the compensator chooses an appropriate quantized control input from the received packet and applies it to the plant.

The discrete-time system studied in this paper is described by

$$P : \begin{cases} x(k+1) = Ax(k) + Bv^*(k) \\ y(k) = Cx(k), \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$, $v^* \in \mathbf{R}^l$, and $y \in \mathbf{R}^p$ are state vector, control input, and system output, respectively. $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times l}$, and $C \in \mathbf{R}^{p\times n}$ are system matrices. The initial state is $x_0 \in \mathbf{R}^n$.

The dynamic quantizer Q^* $(i = 0, 1, 2, \dots, N)$ is given by

$$Q^* : \begin{cases} \xi(k+1) \\ \quad = \mathscr{A}\xi(k) + \mathscr{B}(v(k) - u(k)) \\ \xi(k+2 \mid k) \\ \quad = \mathscr{A}\xi(k+1 \mid k) \\ \qquad + \mathscr{B}(v(k+1 \mid k) - u(k+1 \mid k)) \\ \quad \vdots \\ \xi(k+N+1 \mid k) \\ \quad = \mathscr{A}\xi(k+N \mid k) \\ \qquad + \mathscr{B}(v(k+N \mid k) - u(k+N \mid k)) \\ v(k) = q_\mu(\mathscr{C}\xi(k) + u(k)) \\ v(k+1 \mid k) = q_\mu(\mathscr{C}\xi(k+1 \mid k) + u(k+1 \mid k)) \\ \quad \vdots \\ v(k+N \mid k) \\ \quad = q_\mu(\mathscr{C}\xi(k+N \mid k) + u(k+N \mid k)), \end{cases} \tag{2}$$

where $\xi(k+1) = \xi(k+1 \mid k)$, $\xi \in \mathbf{R}^n$, $u^* \in \mathbf{R}^l$, and $v \in \mathbf{V}^l$ are the state, input, and output of Q_i^* ($\mathbf{V} \subset \mathbf{R}$ are quantization sets of Q_i^*) and $\mathscr{A}$, $\mathscr{B}$, and $\mathscr{C}$ are system matrices that will be designed in Section 4. Set the initial states $\xi(0) = 0$ and $v(k) = 0$ when $u(k) = 0$.

The static part q_μ of (2) can be obtained as [15]

$$q_\mu(x) = \begin{cases} \mu(k) M, & \text{if } \dfrac{x}{\mu(k)} > M - \dfrac{\Delta}{2} \\ -\mu(k) M, & \text{if } \dfrac{x}{\mu(k)} \le -M + \dfrac{\Delta}{2} \\ \left\lfloor \dfrac{x}{\Delta\mu(k)} + \dfrac{1}{2} \right\rfloor \Delta\mu(k), & \text{if } \dfrac{|x|}{\mu(k)} \le M - \dfrac{\Delta}{2}, \end{cases} \tag{3}$$

where Δ is its sensitivity and M is the saturation value. We use $\lfloor a \rfloor$ to represent the biggest integer that satisfies $\lfloor a \rfloor \le a$ in our paper. $\mu(k)$ is the scaling factor that is monotonically nonincreasing, which will be considered later in our paper.

Moreover, the following assumptions are made in this paper.

Assumption 1. Consider $l = p$ (the dimensions of v and y are the same) and the matrix CB is nonsingular.

Assumption 2. (A, B) is controllable and (A, C) is observable.

Assumption 3. The delay in the forward channel satisfies $0 \le d(k) \le d_m$.

Assumption 4. The maximum consecutive number of packet loss in the forward channel is p_m.

3. The Predictive Compensation Strategy

To compensate time-varying delay and packet loss in our system, we introduce the predictive compensate controlstrategy

[1, 2] in our paper, which is composed of an observer, a prediction generator, and a delay compensator. The idea of our NQPC method is that all the possible future control inputs are quantized and packed into a packet before transmission, and then the compensator chooses an appropriate control input from the received packet and applies it to the plant.

Firstly a state observer can be given as

$$\hat{x}(k+1 \mid k) = A\hat{x}(k \mid k-1) + Bu(k) + L[y(k) - C\hat{x}(k \mid k-1)], \quad (4)$$

where $\hat{x}(k+1 \mid k) \in \mathbf{R}^n$ is the one step state prediction, $u(k)$ is the input, and L is the system matrix.

For the quantized predictive control system, it is clear that the length of predictive sequence must be equal to or bigger than upper bound of the total network induced delay and packet loss. Therefore, we have integer N satisfying $N \geq d_m + p_m$ which means the prediction is able to compensate for delay and pack loss in the forward channel and N is the length of the predictive sequence.

Based on (4) and the output data up to k, state predictive sequence from instant $k+1$ to $k+N$ can be constructed as

$$\begin{aligned}
\hat{x}(k+1 \mid k) &= A\hat{x}(k \mid k-1) + Bu(k) \\
&\quad + L[y(k) - C\hat{x}(k \mid k-1)] \\
\hat{x}(k+2 \mid k) &= A\hat{x}(k+1 \mid k) + Bu(k+1 \mid k) \\
\hat{x}(k+3 \mid k) &= A\hat{x}(k+2 \mid k) + Bu(k+2 \mid k) \\
&\vdots \\
\hat{x}(k+N \mid k) &= A\hat{x}(k+N-1 \mid k) + Bu(k+N-1 \mid k)
\end{aligned} \quad (5)$$

with

$$u(k+i \mid k) = K\hat{x}(k+i \mid k), \quad (6)$$

where, $i = \{1, 2, \ldots, N\}$, $K \in \mathbf{R}^{l\times n}$ is the state feedback gain, and the way we choose K is the same as that of traditional control systems.

This results in

$$\begin{aligned}
\hat{x}(k+i \mid k) &= (A+BK)^{i-1}\hat{x}(k+1 \mid k) \\
&= (A+BK)^{i-1}[(A+BK-LC)\hat{x}(k \mid k-1) + LCx(k)],
\end{aligned} \quad (7)$$

where $i = \{1, 2, \ldots, N\}$.

In this paper, output of the control prediction generator at instant k can be given as

$$\left[u(k)^T \; u(k+1 \mid k)^T \; u(k+2 \mid k)^T \; \cdots \; u(k+N \mid k)^T\right]^T. \quad (8)$$

Remark 5. It is clear in (8) that output sequence length of the control prediction generator is $N+1$, which means that the sequence is composed of two parts: the real-time control part $u(k)$ and the predictive control part $[u(k+1 \mid k)^T \; u(k+2 \mid k)^T \; \cdots \; u(k+N \mid k)^T]^T$. When a control output is transmitted without suffering delay or packet loss, the real-time control part $u(k)$ will be used for control. When delay or packet loss occurs during transmission, the predictive control part will be used for control.

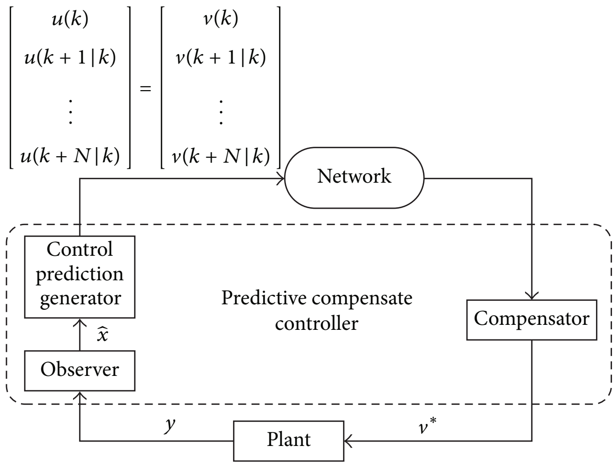

Figure 2: Unquantized predictive control system.

Output of the dynamic quantizer Q^* at instant k is a sequence

$$\begin{aligned}
v(k) &= q_\mu(\mathscr{C}\xi(k) + u(k)) \\
v(k+1 \mid k) &= q_\mu(\mathscr{C}\xi(k+1 \mid k) + u(k+1 \mid k)) \\
&\vdots \\
v(k+N \mid k) &= q_\mu(\mathscr{C}\xi(k+N \mid k) + u(k+N \mid k)),
\end{aligned} \quad (9)$$

where $v(k+i \mid k)$ $(i = \{0, 1, 2, \ldots, N\})$ is the quantized output signal obtained by quantizer Q^*.

In this paper both packet loss and time-varying delay are considered as delay. Define a bounded random scalar $0 \leq \tau(k) \leq d_m + p_m$. As is depicted in Figure 1, since the quantizer output $v(k)$ is transmitted through the network with delay and packet loss, let $v(k - \tau(k))$ denote the delayed quantizer output received by the compensator at instant k.

For the system considered in this paper, since more than one predictive sequence may arrive at the compensator side at the same time, assume that only the newest predictive sequence is used at each instant.

Then output of the delay compensator at instant k can be obtained as

$$\begin{aligned}
v^*(k) &= v(k \mid k - \tau(k)) \\
&= q_\mu[\mathscr{C}\xi(k \mid k-\tau(k)) + u(k \mid k-\tau(k))].
\end{aligned} \quad (10)$$

4. Design of a Dynamic Quantizer

In this section, parameters of the dynamic quantizer (2) are designed.

The system in Figure 2 is an unquantized nominal system, where the initial state, plant, controller, predictive strategy, delay, and packet losses process of that system are same as those in the quantized system in Figure 1.

Definition 6. Define the maximum output error between the two systems as

$$\mathrm{Er}\left(Q^{*}\right)=\max_{k\in\mathbf{Z}_{+}}\left\|y\left(k,x_{0}\right)-y^{*}\left(k,x_{0}\right)\right\|,\tag{11}$$

where for instance k, $y(k)$ is output of the NQPC system in Figure 1 and $y^{*}(k)$ is output of the nominal system in Figure 2 and the initial state is $x(0)=x_0$.

Remark 7. $\mathrm{Er}(Q^{*})$ in (11) represent the output difference between the NQPC system and its nominal system. The upper bound of $\mathrm{Er}(Q^{*})$ is minimized through parameters redesigning of (2), which is an optimal approximation of the nominal system in the sense of input-output relation.

Through parameters redesigning, a dynamic quantizer for our NQPC system can be given according to the following theorem.

Theorem 8. *The dynamic quantizer of the NQPC system that is able to minimize the quantized error can be given as*

$$Q^{*}:\begin{cases}\xi\left(k+1\right)=A\xi\left(k\right)B\left(v\left(k\right)-u\left(k\right)\right)\\ \xi\left(k+2\mid k\right)\\ \quad =A\xi\left(k+1\mid k\right)\\ \quad\quad +B\left(v\left(k+1\mid k\right)-u\left(k+1\mid k\right)\right)\\ \quad\vdots\\ \xi\left(k+N+1\mid k\right)\\ \quad =A\xi\left(k+N\mid k\right)\\ \quad\quad +B\left(v\left(k+N\mid k\right)-u\left(k+N\mid k\right)\right)\\ v\left(k\right)=q_{\mu}\left(C_{Q}\xi\left(k\right)+u\left(k\right)\right)\\ v\left(k+1\mid k\right)=q_{\mu}\left(C_{Q}\xi\left(k+1\mid k\right)+u\left(k+1\mid k\right)\right)\\ \quad\vdots\\ v\left(k+N\mid k\right)\\ \quad =q_{\mu}\left(C_{Q}\xi\left(k+N\mid k\right)+u\left(k+N\mid k\right)\right)\end{cases}\tag{12}$$

and the upper bound of the maximum output error can be minimized by

$$Er\left(Q^{*}\right)\leq\|CB\|\frac{\Delta\mu\left(0\right)}{2},\tag{13}$$

where $C_Q=-(CB)^{-1}CA$.

Proof. For the NQPC system described in previous section, from (1), (2), (6), and (9) we have the following equations:

$$\begin{aligned}u\left(k\right)&=u\left(k\mid k-\tau\left(k\right)\right)\\&=K\left(A+BK\right)^{\tau(k)-1}\left[\left(A+BK-LC\right)\hat{x}\right.\\&\qquad\times\left(k-\tau\left(k\right)\mid k-\tau\left(k\right)-1\right)\\&\qquad\left.+LCx\left(k-\tau\left(k\right)\right)\right],\\ x\left(k+1\right)&=Ax\left(k\right)+Bv^{*}\left(k\right)\\&=Ax\left(k\right)+B\mathscr{C}\xi\left(k\mid k-\tau\left(k\right)\right)+B\omega\left(k\right)\\&\quad+Bu\left(k\mid k-\tau\left(k\right)\right)\\&=Ax\left(k\right)+B\mathscr{C}\xi\left(k\mid k-\tau\left(k\right)\right)+B\omega\left(k\right)\\&\quad+BK\left(A+BK\right)^{\tau(k)-1}\\&\quad\times\left[\left(A+BK-LC\right)\hat{x}\left(k-\tau\left(k\right)\mid k-\tau\left(k\right)-1\right)\right.\\&\qquad\left.+LCx\left(k-\tau\left(k\right)\right)\right],\\ \hat{x}\left(k+1\mid k\right)&=A\hat{x}\left(k\mid k-1\right)+Bu\left(k\right)\\&\quad+L\left[y\left(k\right)-C\hat{x}\left(k\mid k-1\right)\right]\\&=\left(A+BK-LC\right)\hat{x}\left(k\mid k-1\right)+LCx\left(k\right),\\ \xi\left(k+1\right)&=\mathscr{A}\xi\left(k\right)+\mathscr{B}\left(v\left(k\right)-u\left(k\right)\right)\\&=\left(\mathscr{A}+\mathscr{B}\mathscr{C}\right)\xi\left(k\right)+\mathscr{B}\omega\left(k\right)\\ \xi\left(k+1\mid k\right)&=\left(\mathscr{A}+\mathscr{B}\mathscr{C}\right)\xi\left(k\right)+\mathscr{B}\omega\left(k\right)\\ \xi\left(k+1\mid k-1\right)&=\left(\mathscr{A}+\mathscr{B}\mathscr{C}\right)\xi\left(k\mid k-1\right)+\mathscr{B}\omega\left(k\right),\\&\vdots\\ \xi\left(k+1\mid k-N+1\right)&\\&=\left(\mathscr{A}+\mathscr{B}\mathscr{C}\right)\xi\left(k\mid k-N+1\right)+\mathscr{B}\omega\left(k\right),\end{aligned}\tag{14}$$

where $\omega(k)=q_{\mu}(\mathscr{C}\xi(k\mid\cdot)+u(k))-(\mathscr{C}\xi(k\mid\cdot)+u(k))$. □

Letting $S=(A+BK)$,

$$\begin{aligned}&z\left(k\right)\\&=\left[\,x\left(k\right)^{T}\;\;x\left(k-1\right)^{T}\;\cdots\;x\left(k-N\right)^{T}\;\;\hat{x}\left(k\mid k-1\right)^{T}\;\;\hat{x}\left(k-1\mid k-2\right)^{T}\;\cdots\;\hat{x}\left(k-N\mid k-N-1\right)^{T}\right.\\&\quad u\left(k-1\right)^{T}\;\;u\left(k-2\right)^{T}\;\cdots\;u\left(k-N\right)^{T}\;\;\xi\left(k\right)^{T}\xi\left(k-1\right)^{T}\;\cdots\;\xi\left(k-N\right)^{T}\;\;\xi\left(k\mid k-1\right)^{T}\\&\quad\xi\left(k-1\mid k-2\right)^{T}\;\cdots\;\xi\left(k-N+1\mid k-N\right)^{T}\;\;\xi\left(k\mid k-2\right)^{T}\;\;\xi\left(k-1\mid k-3\right)^{T}\;\cdots\;\xi\left(k-N+2\mid k-N\right)^{T}\\&\quad\left.\cdots\;\;\xi\left(k\mid k-N\right)^{T}\,\right]^{T},\end{aligned}$$

$$\overline{B}=\left[B^{T}\;\;0^{T}_{l\times[n(2N+1)+lN]}\;\;\mathscr{B}^{T}\;\;0^{T}_{l\times nN}\;\;\mathscr{B}^{T}\;\;0^{T}_{l\times n(N-1)}\;\cdots\;\mathscr{B}^{T}\right]^{T},$$

$$\overline{C}=\left[C\;\;0_{p\times[n(3N+2+N(N+1)/2)+lN]}\right],$$

$$\Xi_{\sigma(k)} = \begin{bmatrix} \prod_{11} & \prod_{12} & 0 & \prod_{14} \\ \prod_{21} & \prod_{22} & 0 & 0 \\ \prod_{31} & \prod_{32} & \prod_{33} & 0 \\ 0 & 0 & 0 & \prod_{44} \end{bmatrix},$$

$$\prod_{11} = \begin{bmatrix} A & 0_{n\times n(\tau(k)-1)} & BKS^{\tau(k)-1}LC & 0_{n\times n(N-\tau(k))} \\ I_{nN\times nN} & 0_{nN\times n} & & \end{bmatrix},$$

$$\prod_{12} = \begin{bmatrix} 0_{n\times n\tau(k)} & BKS^{\tau(k)-1}\left(A+BK-LC\right) & 0_{n\times n(N-\tau(k))} \\ 0_{nN\times n(N+1)} & & \end{bmatrix},$$

$$\prod_{14} = \begin{bmatrix} 0_{n\times n[(\tau(k)+1)N-\tau(k)(\tau(k)-1)/2]} & \mathscr{B}\mathscr{C} & 0_{n\times n(N(N+1)/2-\tau(k)N+\tau(k)(\tau(k)-1)/2-1)} \\ 0_{nN\times n(N(N+1)/2)} & & \end{bmatrix},$$

$$\prod_{21} = \begin{bmatrix} LC & 0_{n\times nN} \\ 0_{nN\times n(N+1)} & \end{bmatrix},$$

$$\prod_{22} = \begin{bmatrix} A-LC & 0_{n\times n(\tau(k)-1)} & BKS^{\tau(k)-1}\left(A-LC\right) & 0_{n\times n(N-\tau(k))} \\ I_{nN\times nN} & 0_{nN\times n} & & \end{bmatrix},$$

$$\prod_{31} = \begin{bmatrix} 0_{l\times n\tau(k)} & KS^{\tau(k)-1}LC & 0_{l\times n(N-\tau(k))} \\ 0_{l(N-1)\times n(N+1)} & & \end{bmatrix},$$

$$\prod_{32} = \begin{bmatrix} 0_{l\times n\tau(k)} & KS^{\tau(k)-1}\left(A+BK-LC\right) & 0_{l\times n(N-\tau(k))} \\ 0_{l(N-1)\times n(N+1)} & & \end{bmatrix},$$

$$\prod_{33} = \begin{bmatrix} 0_{l\times lN} & \\ I_{l(N-1)\times l(N-1)} & 0_{l(N-1)\times l} \end{bmatrix},$$

$$\prod_{44} = \begin{bmatrix} \mathscr{A}+\mathscr{B}\mathscr{C} & \overbrace{0_{n\times n} \quad\quad\quad\quad \cdots \quad\quad 0_{n\times n}}^{\frac{N(N+3)}{2}} & & & & \\ I_{nN\times nN} & 0_{nN\times n} & & \cdots & & 0_{nN\times n} \\ \mathscr{A}+\mathscr{B}\mathscr{C} & 0_{n\times n} & & \cdots & & 0_{n\times n} \\ 0_{n(N-1)\times n} & I_{n(N-1)\times n(N-1)} & 0_{n(N-1)\times n} & \cdots & & 0_{n(N-1)\times n} \\ 0_{n\times n} & \mathscr{A}+\mathscr{B}\mathscr{C} & 0_{n\times n} & \cdots & & 0_{n\times n} \\ 0_{n(N-2)\times n} & 0_{n(N-2)\times n} & I_{n(N-2)\times n(N-2)} & 0_{n(N-2)\times n} & \cdots & 0_{n(N-2)\times n} \\ \vdots & & \ddots & & & \\ 0_{n\times n} & \cdots & 0_{n\times n} & \mathscr{A}+\mathscr{B}\mathscr{C} & & 0_{n\times nN} \\ 0_{n\times n} & \cdots & 0_{n\times n} & 0_{n\times n} & & I_{nN\times nN} \\ 0_{n\times n} & \cdots & 0_{n\times n} & 0_{n\times n} & & \mathscr{A}+\mathscr{B}\mathscr{C} \end{bmatrix}. \quad (15)$$

Then the NQPC system can be written as the following switched system:

$$z(k+1) = \Xi_{\sigma(k)} z(k) + \overline{B}\omega(k)$$
$$y(k) = \overline{C} z(k), \tag{16}$$

where the state matrix $\Xi_{\sigma(k)}$ switches in the set of possible matrices $\{\Xi_0 \ \cdots \ \Xi_N\}$ according to the parameter $\sigma(k)$ called the switching function, which takes value from the finite index set $\mathbf{F} = \{0, 1, 2, \ldots, N\}$.

Moreover, the nominal system in Figure 2 is given as

$$\xi(k+1) = \Phi_{\sigma(k)} \xi(k)$$
$$y^*(k) = \widehat{C}\xi(k), \tag{17}$$

where

$$\xi_k = [\, x(k)^T \ \ x(k-1)^T \ \cdots \ x(k-N)^T \ \ \hat{x}(k \mid k-1)^T$$
$$\hat{x}(k-1 \mid k-2)^T \ \cdots \ \hat{x}(k-N \mid k-N-1)^T$$
$$u(k-1)^T \ \ u(k-2)^T \ \cdots \ u(k-N)^T \,]^T,$$
$$\Phi_{\sigma(k)} = \begin{bmatrix} \prod_{11} & \prod_{12} & 0 \\ \prod_{21} & \prod_{22} & 0 \\ \prod_{31} & \prod_{32} & \prod_{33} \end{bmatrix}, \qquad \widehat{C} = [C \ \ 0_{p\times n(3N+1)}]. \tag{18}$$

Therefore, difference between $y^*(S, x_0)$ and $y(S, x_0)$ ($S \in \mathbf{Z}_+$) is

$$\begin{aligned} & y^*(S, x_0) - y(S, x_0) \\ & = \widehat{C}\Phi^S_{\sigma(k)} \begin{bmatrix} x_0 \\ 0_{n(4N+2)\times 1} \end{bmatrix} - \overline{C}\Xi^S_{\sigma(k)} \begin{bmatrix} x_0 \\ 0_{n(3N+1)\times 1} \end{bmatrix} \\ & \quad - \overline{C}\sum_{l=0}^{S-1} \Xi_{\sigma(k)}^{(S-1)-l} \overline{B}\omega(l) \\ & = -\overline{C}\sum_{l=0}^{S-1} \Xi_{\sigma(k)}^{(S-1)-l} \overline{B}\omega(l). \end{aligned} \tag{19}$$

It is clear that

$$\begin{aligned} \|y^*(S, x_0) - y(S, x_0)\| & \le \left\| \sum_{l=0}^{S-1} \overline{C}\Xi_{\sigma(k)}^{(S-1)-l}\overline{B} \right\| \frac{\Delta\mu(0)}{2} \\ & = \left\| CB + \sum_{l=1}^{S-1} \overline{C}\Xi_{\sigma(k)}^{(S-1)-l}\overline{B} \right\| \frac{\Delta\mu(0)}{2}. \end{aligned} \tag{20}$$

Moreover, when $\mathscr{A} = A$, $\mathscr{B} = B$, and $\mathscr{C} = C_Q$, we can get $\overline{C}\Xi_{\sigma(k)}^{(S-1)-l}\overline{B} = 0$ ($l = 1, 2, \ldots, \infty$), which means that the latter part of (20) is minimized, and therefore it can be given that

$$\mathrm{Er}(Q^*) \le \|CB\| \frac{\Delta\mu(0)}{2}. \tag{21}$$

As a result, upper bound of the maximum output error $\mathrm{Er}(Q^*)$ between the NQPC system and its nominal system is minimized, the dynamic quantizer is given in (12), and the smallest upper bound of $\mathrm{Er}(Q^*)$ is obtained in (21).

Remark 9. In this paper the dynamic quantizer (12) is different from that in [17]. Since the scaling quantizer q_μ (3) is used for dynamic quantization instead of traditionally static quantizer in [17], improved dynamic quantizers are obtained for our system, which can finally eliminate the quantized error by adjusting the parameter $\mu(k)$. The adjustment procedure of $\mu(k)$ will be considered in the proof of Theorem 11.

5. Stability Analysis

In this section, a sufficient condition for stability is obtained for the NQPC system, and the way dynamic quantizer works is explained.

Firstly, we have the following lemma.

Lemma 10. *System state $z(k)$ that starts from region R_1 described in (30) will enter region R_{i+1} in $i\pi$ steps, where R_{i+1} can be given by*

$$R_{i+1} = \left\{ z(k) : z^T(k) P z(k) \le \mu(k)^2 \frac{M^2}{\|\mathcal{F}\|^2} \lambda_{\min}(P) \right\}. \tag{22}$$

Proof of Lemma 10 is in the appendix, and $R_1, \pi, \mu(k)$, and $\mathcal{F}$ are defined in the proof of Theorem 11.

Then, main result of our paper is presented by the following theorem.

Theorem 11. *For NQPC system (16) with time-varying delay and packet loss in the forward channel, it is asymptotically stable if there exist $P = P^T > 0$ and $\alpha > 0$ satisfying that*

$$(1+\alpha)\Xi_i^T P \Xi_i - P < 0, \tag{23}$$

where $i = \{0, 1, 2, \ldots, N\}$.

Proof. Firstly, let $V(k) = z^T(k) P z(k)$, where $P \in \mathbf{R}^{[n(3N+3+N(N+1)/2)+lN]\times[n(3N+3+N(N+1)/2)+lN]}$ is a positive definite matrix, and $\Delta V(k)$ can be obtained as

$$\begin{aligned} \Delta V(k) & = V(k+1) - V(k) \\ & = z^T(k+1) P z(k+1) - z^T(k) P z(k) \\ & = z^T(k) \left(\Xi^T_{\sigma(k)} P \Xi_{\sigma(k)} - P \right) z(k) \\ & \quad + 2z^T(k) \Xi^T_{\sigma(k)} P \overline{B}\omega(k) + \omega^T(k) \overline{B}^T P \overline{B}\omega(k) \\ & \le z^T(k) \left[(1+\alpha) \Xi^T_{\sigma(k)} P \Xi_{\sigma(k)} - P \right] z(k) \\ & \quad + \left(1+\alpha^{-1}\right) \omega^T(k) P \omega(k) \\ & \le -\left[\lambda_{\min}(D_i) |z(k)|^2 + \left(1+\alpha^{-1}\right) \left\| \overline{B}^T P \overline{B} \right\| \Delta^2\mu^2(k) \right] \\ & \le -\left[\lambda_{\min}(D) |z(k)|^2 + \left(1+\alpha^{-1}\right) \left\| \overline{B}^T P \overline{B} \right\| \Delta^2\mu^2(k) \right], \end{aligned} \tag{24}$$

where α is a positive scalar, $D_i = -[(1+\alpha)\Xi_i^T P \Xi_i - P]$ is assumed to satisfy $D_i > 0$ for $i = \{0, 1, 2, \ldots, (N_1+1)(N_2+1)-1\}$, and $\lambda_{\min}(D) = \min[\lambda_{\min}(D_i)]$, where $\lambda_{\min}(D_i)$ denotes the smallest eigenvalue of D_i.

It is clear that $\Delta V(k) < 0$ when Theorem 11 is satisfied, and the state of (16) outside the region

$$H = \{z(k) : |z(k)| \leq \Theta \Delta \mu(k)\} \tag{25}$$

will ultimately converge to H, where $\Theta = \sqrt{(1+\alpha^{-1})\|\overline{B}^T P \overline{B}\| / \lambda_{\min}(D)}$.

To ensure asymptotic stability of the system (16) inside region H, the "zoom" method proposed in [15, 16] is used for the following proof.

The zooming-in stage is as follows.

Rewrite $\omega(k)$ as $\omega(k) = q_\mu(\digamma z(k)) - \digamma z(k)$, where

$$\digamma = \begin{bmatrix} K & 0_{p\times n(3N+1+\tau(k)(N+1)-\tau(k)(\tau(k)-1)/2)} & C_Q & 0_{p\times n(N(N+3)/2-\tau(k)(N+1)-\tau(k)(\tau(k)-1)/2)} \end{bmatrix}. \tag{26}$$

Let $\mu(k) = \beta^k$, where β is a given constant satisfying $\beta < 1$. Then $\mu(k)$ can be initialized as

$$\mu(0) = \|A\|^\gamma, \tag{27}$$

where $\gamma = \min\{k \geq 1 : \|q_\mu(\digamma z(k)/\mu(k))\| \leq M\sqrt{\lambda_{\min}(P)/\lambda_{\max}(P)} - \Delta/2\}$.

It follows that

$$\left\| \frac{\digamma z(k)}{\mu(0)} \right\| \leq \left\| q_\mu \left(\frac{\digamma z(k)}{\mu(0)} \right) \right\| + \frac{\Delta}{2} \leq M \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}. \tag{28}$$

Then we can obtain that

$$|z_k| \leq \frac{M\mu(0)}{\|\digamma\|} \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}. \tag{29}$$

Therefore, we can get that the initial state $z(0)$ belongs to the region

$$R_1 = \left\{ z(k) : z^T(k) P z(k) \leq \frac{M^2}{\|\digamma\|^2} \mu^2(0) \lambda_{\min}(P) \right\}. \tag{30}$$

Define the scaling factor χ as

$$\chi = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \sqrt{\Theta^2 + \varepsilon} \|\digamma\| \Delta M^{-1}, \tag{31}$$

where $\varepsilon > 0$ is a fixed constant. Choose M, Δ in (31) to ensure $\chi < 1$, and it is clear that $R_1 \supset H$, and $z(k)$ will never leave R_1.

The zooming-out stage is as follows. Define

$$\widehat{\pi} = \frac{M^2 \lambda_{\min}(P)}{\|\digamma\|^2 \lambda_{\min}(D) \Delta^2 \varepsilon} - \frac{\Theta^2 \lambda_{\max}(P)}{\lambda_{\min}(D) \varepsilon},$$
$$\mu(k) = \chi^{\lfloor k/\pi \rfloor} \mu(0), \tag{32}$$

where $k \geq 0$ and $\pi = \lfloor \widehat{\pi} \rfloor$, and it is clear that $\widehat{\pi} > 0$ as $\chi < 1$.

From Lemma 10, we can obtain that the system state $z(k)$ belongs to R_2 for $k \geq \pi$

$$R_2 = \left\{ z(k) : z^T(k) P z(k) \leq (\chi\mu(0))^2 \frac{M^2}{\|\digamma\|^2} \lambda_{\min}(P) \right\}, \tag{33}$$

where $\mu(k) = \chi\mu(0)$ for $k \geq \pi$.

It is clear that the radius of region R_2 becomes smaller than R_1, which means that the state of the system converges after π steps from the initial state.

Similarly, we have the system state $z(k)$ belonging to R_{i+1} for $k \geq i\pi$

$$R_{i+1} = \left\{ z(k) : z^T(k) P z(k) \leq (\chi^i \mu(0))^2 \frac{M^2}{\|\digamma\|^2} \lambda_{\min}(P) \right\}, \tag{34}$$

where $\mu(k) = \chi^i \mu(0)$ for $k \geq \pi$.

As a result, it can be obtained that $\mu(k) \to 0$ when $k \to \infty$ and $\lim_{k\to\infty} |z(k)| = 0$ as the radius of R_n $(n \to \infty)$ goes to 0, and then the proof of Theorem 11 is completed. □

6. Simulation and Practical Examples

In this section, both simulation and practical examples are given to illustrate the advantages of the proposed method.

6.1. Simulation Example. Consider the discrete-time plant described by

$$A = \begin{bmatrix} 0.9994 & 0.0096 \\ -0.1125 & 0.9154 \end{bmatrix}, \quad B = \begin{bmatrix} 4.8563 \times 10^{-5} \\ 0.0096 \end{bmatrix},$$
$$C = \begin{bmatrix} 17630 & 11840 \end{bmatrix} \tag{35}$$

which is controlled over the network, and matrices A, B, and C are obtained through system identification of the DC motor with sampling period 10 ms. Set $N = 9$, which means that the maximum delay in the forward channel is 90 ms. The networked delay in our simulation is shown in Figure 3.

Choose K and L to be

$$K = \begin{bmatrix} -6.75 & -4.784 \end{bmatrix},$$
$$L = \begin{bmatrix} 1.763 \times 10^{-6} & 1.184 \times 10^{-6} \end{bmatrix}. \tag{36}$$

There exist positive scalars $\alpha = 0.01$, positive definite matrix P, and quantizer parameters $\Delta = 0.05$, $M = 6$ satisfying Theorem 8.

Let $\Omega = 0.8$, $\tau = 80$, and give a step input to the plant at $t = 1$ s. Then simulation results of the system can be shown in Figure 4, where the proposed quantized predictive control method is compared with six other methods: the predictive

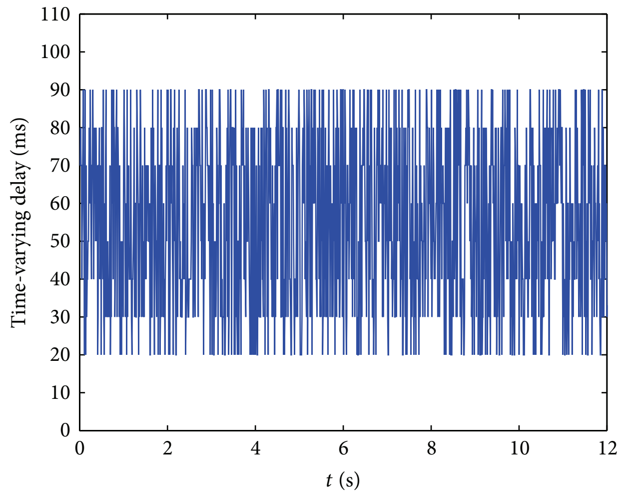

Figure 3: Networked delay in simulation.

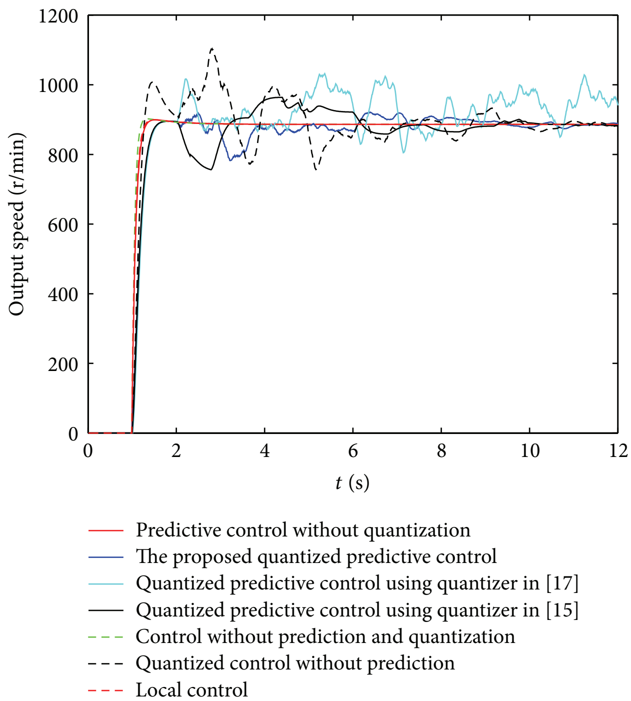

Figure 4: Outputs of the system in simulation.

method without quantization, the predictive method using quantizer in [17], the predictive method using quantizer in [15], networked method without prediction and quantization, the proposed quantized method without prediction, and local method. It is clear in Figure 4 that proposed quantized predictive control method is better than other quantized predictive control methods and is more similar to local control, which means that the proposed method is able to compensate the networked delay well.

6.2. Experimental Example. A test rig was built in our lab to test the proposed method, whose experimental diagram is given in Figure 5. Signals were sent from the control box (Figure 8) to the actuator with the help of the trans-

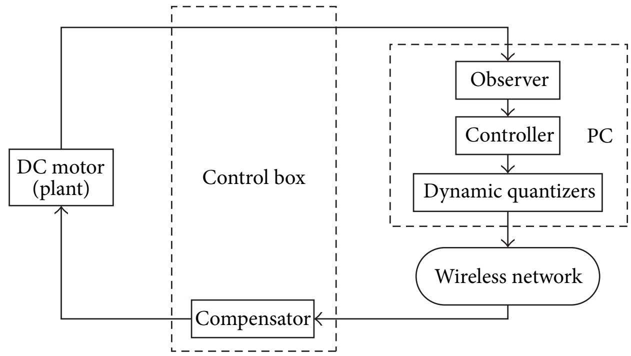

Figure 5: Experimental diagram.

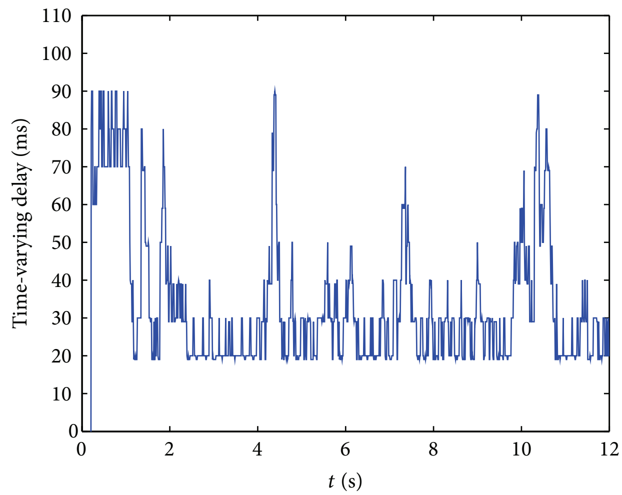

Figure 6: Networked delay in experiment.

mission board (Figure 9). The control box's IP address is 192.168.100.20, and the computer's IP address is 192.168.100.21. They communicate with each other by Wi-Fi IEEE 802.11b.

In our test rig, a DC motor (Figure 10) with sampling period 10 ms is controlled, which can be given by (35). Matrices K, L, P, and α and quantizer parameters Δ, M, Ω, and τ are the same as those in simulation example section.

To illustrate the effectiveness of the proposed method, seven cases are studied. Firstly, predictive control without quantization is studied. Then the proposed quantized predictive control method is studied, where networked delay varies between 0 s and 0.18 s as shown in Figure 6. Compared with the proposed method, quantized predictive control methods using quantizer in [15, 17] are studied. Meanwhile, networked control without prediction, quantized control without prediction, and local control are studied. Results of the experiments are given in Figure 7, and it is clear that proposed method performs well and its output of DC motor is very close to that of the local control method.

7. Conclusion

The design and stability of networked quantized predictive control systems where time-varying delay and packet loss

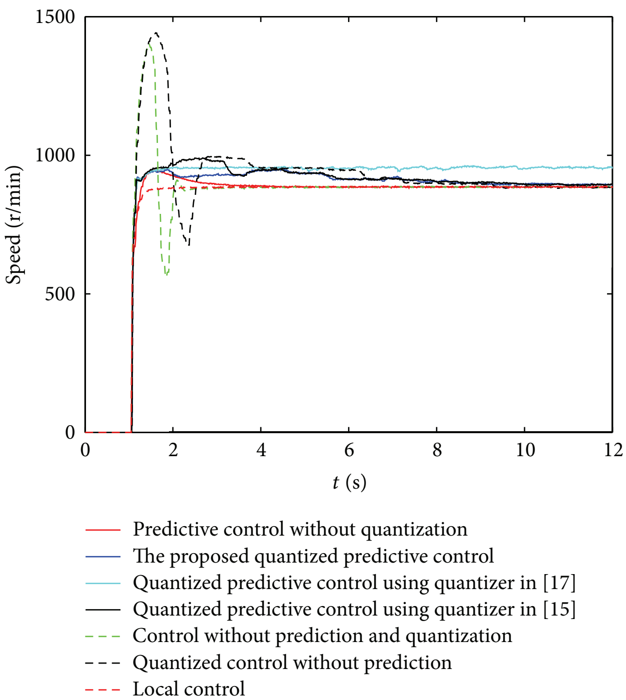

Figure 7: Outputs of the system in experiment.

Figure 8: The control box.

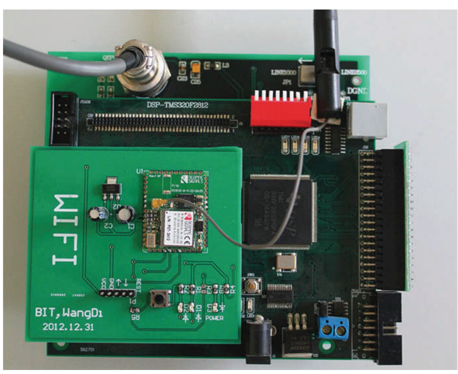

Figure 9: The transmission board.

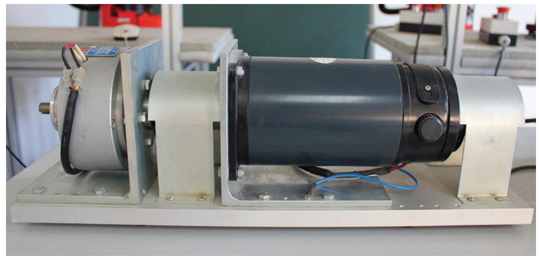

Figure 10: The DC motor.

occur in the forward channel have been investigated in this paper. Based on predictive control method, a model that considers delay, packet loss, and optimal quantization has been analyzed. By redesigning the original dynamic quantizer, a dynamic quantizer that can minimize output error of our system is obtained. The stability problem of the given NQPC system has been transformed into stability of a switched system, and a sufficient condition has been presented. Finally, effectiveness of our method has been shown by both simulation and experimental examples.

Appendix

Proof of Lemma 10. Assume the system state at instant π satisfying that

$$z^T(\pi) Pz(\pi) \leq \Delta^2 \mu^2(0) \left(\Theta^2 + \varepsilon\right) \lambda_{\max}(P). \tag{A.1}$$

If inequality (A.1) is not true, then we have

$$z^T(\pi) Pz(\pi) > \Delta^2 \mu^2(0) \left(\Theta^2 + \varepsilon\right) \lambda_{\max}(P) \tag{A.2}$$

which means that $|z(\pi)|^2 > \Delta^2\mu^2(0)(\Theta^2+\varepsilon)$ for all $k \in [0,\pi]$.

From (30) and (A.2) we can obtain that

$$\begin{aligned} & z^T(\pi) Pz(\pi) - z^T(0) Pz(0) \\ & \quad \geq \Delta^2 \mu^2(0) \left(\Theta^2 + \varepsilon\right) \lambda_{\max}(P) - \frac{M^2}{\|F\|^2} \mu^2(0) \lambda_{\min}(P) \\ & \quad = \lambda_{\max}(P) \Theta^2 \Delta^2 \mu^2(0) - \frac{M^2}{\|F\|^2} \mu^2(0) \lambda_{\min}(P). \end{aligned} \tag{A.3}$$

Nevertheless, from (24), (A.1), and $\chi < 1$, the following inequality can be obtained:

$$\begin{aligned} & \Delta V(\pi - 1) \\ & \quad = z^T(\pi) Pz(\pi) - z^T(\pi - 1) Pz(\pi - 1) \\ & \quad \leq -\lambda_{\min}(D) |z(\pi - 1)|^2 + \lambda_{\min}(D) \Theta^2 \Delta^2 \mu^2(0) \\ & \quad < -\lambda_{\min}(D) \Delta^2 \mu^2(0) \varepsilon. \end{aligned} \tag{A.4}$$

Similarly, it can be obtained that

$$\begin{aligned}\Delta V(\pi - j) &= z^T(\pi - j + 1) Pz(\pi - j + 1) - z^T(\pi - j) Pz(\pi - j) \\ &\leq -\lambda_{\min}(D)\left|z(\pi - j)\right|^2 + \lambda_{\min}(D)\Theta^2\Delta^2\mu^2(0) \\ &< -\lambda_{\min}(D)\Delta^2\mu^2(0)\varepsilon,\end{aligned} \tag{A.5}$$

where $j = \{1, 2, 3, \ldots, \pi\}$.

Then we have

$$\begin{aligned}z^T(\pi) Pz(\pi) - z^T(0) Pz(0) &< -\lambda_{\min}(D)\Delta^2\mu^2(0)\varepsilon\pi \\ &\leq -\lambda_{\min}(D)\Delta^2\mu^2(0)\varepsilon\widehat{\pi} \\ &= \lambda_{\max}(P)\Theta^2\Delta^2\mu^2(0) - \frac{M^2}{\|F\|^2}\mu^2(0)\lambda_{\min}(P).\end{aligned} \tag{A.6}$$

As there is a contradiction between (A.3) and (A.6), validity of (A.1) has been proved.

Based on (A.1) and $\chi < 1$, it follows that

$$\begin{aligned}z^T(\pi) Pz(\pi) &\leq \Delta^2\mu^2(0)\left(\Theta^2 + \varepsilon\right)\lambda_{\max}(P) \\ &< \left(\chi\mu(0)\right)^2\frac{M^2}{\|F\|^2}\lambda_{\min}(P).\end{aligned} \tag{A.7}$$

Thus, $z(\pi)$ belongs to

$$R_2 = \left\{z(k) : z^T(k) Pz(k) \leq \left(\chi\mu(0)\right)^2\frac{M^2}{\|F\|^2}\lambda_{\min}(P)\right\}. \tag{A.8}$$

For $\pi \leq k \leq 2\pi$, a similar result can be obtained as

$$z^T(2\pi) Pz(2\pi) \leq \left(\chi^2\mu(0)\right)^2\frac{M^2}{\|F\|^2}\lambda_{\min}(P). \tag{A.9}$$

Moreover, for $(i-1)\pi \leq k \leq i\pi$, it can be obtained that

$$z^T(i\pi) Pz(i\pi) \leq \left(\chi^i\mu(0)\right)^2\frac{M^2}{\|F\|^2}\lambda_{\min}(P), \tag{A.10}$$

which means that the scaling factor $\mu(k)$ is narrowed every π steps. That is, $z(i\pi)$ belongs to

$$R_{i+1} = \left\{z(k) : z^T(k) Pz(k) \leq \left(\chi^i\mu(0)\right)^2\frac{M^2}{\|F\|^2}\lambda_{\min}(P)\right\}. \tag{A.11}$$

The proof of Lemma 10 is completed. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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