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Distributed backstepping-based adaptive fuzzy control of multiple high-order nonlinear dynamics

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Abstract This paper focuses on the cooperative adaptive fuzzy control of multiple high-order nonlinear dynamics. The communication network topology is undirected and fixed. Each individual dynamics is modeled by a high-order integrator incorporating with unknown nonlinear dynamics and an unknown external disturbance. With the approximation capability of fuzzy logic systems, the unknown nonlinear dynamics is compensated by the adaptive fuzzy logic systems scheme. The negative effects of the approximation error and external disturbances are counteracted by employing the robustness terms. Under the backstepping framework, two cooperative adaptive fuzzy controllers are designed for each agent such that all agents ultimately achieve consensus. Moreover, these controllers are distributed in the sense that only the local state information between the agent and its neighbors is required to design the controller. Finally, a simulation example with four-order dynamics demonstrates the effectiveness of the algorithms.

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1 Introduction

Because of better description of the actual physical system, the nonlinear dynamics has been widely concerned [1]. A series of techniques and tools have been developed to address a variety of nonlinear dynamics control problems, and numerous results have been obtained. With advances in technology and industrial needs, the control of a single system has become increasingly unable to meet the needs of many practical engineering applications such as cooperative control of unmanned ground/air/underwater vehicles [2–4], distributed sensor networks [5], aggregation and rendezvous control [6], and attitude alignment of spacecraft [7]. Cooperative control of multiple nonlinear dynamics has received increasing attention by the fact that many benefits can be obtained when a single complicated dynamics is equivalently replaced by multiple simpler ones, especially with lower operational costs, less system requirements, higher robustness, stronger adaptivity, and more flexible scalability. Due to the fact that the single system control scheme cannot make good performance for multiple complex dynamics, decentralized and distributed control scheme has been investigated in the past decade particularly. Among the existing works with respect to multiple system coordination [8–12,30,31], most of them studied only the first- and second-order dynamics. However, in many practical engineering applications, many systems are modeled by higher-order dynamics, such as flexible joint manipulator [1,13–15] and jerk system. It is not acceptable to model the plant dynamics with only single- or double-integrator dynamics. Hence, it is necessary to investigate the coordination problem from lower-order dynamics to higher-order nonlinear ones.

Compared with the first-order and second-order dynamics, the high-order dynamics involve more details related to the interactions between the system dynamics (states and their derivatives) and the communication graph [32]. Besides the aforementioned, in many practical applications, the dynamics of the systems is not only nonlinear, but also with uncertainties, thus solving consensus problems for multiple higher-order uncertain nonlinear dynamics would make great sense for practical applications. Cooperative control of high-order nonlinear systems with uncertainties is more challenging than that of certain high-order linear ones. The extension of adaptive control to high-order nonlinear dynamics is not straightforward because of both the growth of the order and the local information interactions between neighboring dynamics. The challenge is to make sure that both the control protocols and the parameter update laws are distributed. That is, they are only allowed to depend only on locally available information about the current system and its neighbors. High-order systems contain more states and their derivatives, so the design of adaptive control becomes more complicated. This requires the careful crafting of a suitable Lyapunov function, which automatically yields a distributed adaptive controller that depends only on local information.

Because of the inherent characteristics of multiple linear systems, matrix theory approaches are frequently used in stability analysis. In [16], Ren et al. showed a matrix approach-based framework for high-order multi-agent systems. They defined a class of l-order consensus algorithms and showed necessary and sufficient conditions under which each information variable and their higher-order derivatives converged to common values. Jiang and Wang [17] investigated the consensus problem for multi-agent systems with individual agents modeled by high-order integrators under fixed/switching topology and zero/nonzero communication time-delays. In [18], consensus of high-order integrators multi-agent systems with time-delays and switching topologies were studied. Coordination of high-order linear systems with disturbances was investigated in [19]. Discrete-time high-order linear multi-agent systems were considered in [20], and the results for the general high-order linear time-invariant (LTI) systems was published in [21]. Dong et al.[22] considered the tracking control problem. Distributed robust/adaptive control laws were proposed such that the states of each system converged to the desired trajectory asymptotically. However, the model of the systems are without uncertainties. In [23], Dong considered a group of third-order nonlinear systems with parameter uncertainties using backstepping techniques and adaptive control method. But the proposed control law do not work when the order of systems is larger than three. It is worth noting that the control algorithm in [23] cannot easily extend to the high-order nonlinear systems with uncertainties at each order. In [24], high-order nonlinear dynamics tracking control was considered by using the neural network(NN) technique. However, the drawback of [24] is that the NN update laws are using global Laplacian matrix.

In this paper, we tried to solve the high-order nonlinear multi-agent systems control problem under distributed backstepping framework. The key to the design of distributed controller is the selection of a sequence of suitable Lyapunov functions and the adaptive laws that depend on the graph topology and the model of the system. The basis for the selection of suitable graph-dependent Lyapunov functions was laid in the backstepping techniques on the graph. A distributed recursive design approach is proposed to archive consensus of multiple high-order nonlinear systems with uncertainties. The main contributions of this paper include the following: first, this paper reviews the major results and progress in distributed higher-order networked nonlinear dynamics coordination. A kind of practical multiple higher-order nonlinear systems in the Brunovsky form with uncertainties is considered, which include first- and second-order systems as special cases. Second, a systematic controller design procedure is proposed to deal with the control problem through combining distributed backstepping method with adaptive fuzzy control techniques. The adaptive fuzzy control is completely distributed. And the convergence of the system errors is proved rigorously by virtue of the Lyapunov stable theory and Barbalat's lemma.

The subsequent sections are organized as follows: In Sect. 2, the control problem is formally stated and the background as well as necessary preliminaries concerning the control problem are given. In Sect. 3, the cooperative control laws are proposed relying on backstepping method and adaptive fuzzy control approaches. The unknown nonlinear functions are dealt by fuzzy logic systems, and the external disturbances are addressed by applying robust adaptive control method. In Sect. 4, a four-order simulation example is provided to demonstrate the performance of the proposed control laws. The last section concludes this paper.

2 Preliminaries and problem statement

In this section, basic graph theory for networked dynamics, control problem and fuzzy logic systems on graph are introduced.

2.1 Brief graph theory for networked dynamics

With respect to networked dynamics, any control laws must be distributed in the sense that it respects the communication network topology. The communication restrictions by topologies can severely limit the power of local distributed control algorithm at each individual dynamics. The idea of a communication network models the information flows in a multi-agent group. A team of m high-order nonlinear dynamics labeled as system 1 to m are considered. The communication topology among the m systems is assumed to be bidirectional or directional, and the interactions among the nodes are represented by an undirected or directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. The topology $\mathcal{G}$ represents the structure of networked system, where $\mathcal{V}$ is a set of the indices of the systems and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges that describe the communications between the systems. If $(p, j) \in \mathcal{E}$, then p is neighboring to j, meaning system j can obtain information from system p. $\mathcal{A}$ is a weighted adjacency matrix with nonnegative adjacency elements a_{pj}. Moreover, it is assumed that $a_{pp} = 0$. If the state of system p is available to system j, then system p is said to be a neighbor of system j. The neighbor set of node v_j is denoted by $\mathcal{N}_j$, where $j \notin \mathcal{N}_j$ [25].

2.2 Problem statement

In practice, there are several cases where the individual dynamics can be described by a higher-order form. For example, the jerk system is described by third-order differential equations [1]. A single-link flexible joint manipulator can be modeled by a fourth-order nonlinear system. A group of unmanned aerial vehicle formation control problem, in essence, is a high-order multi-agent system network coordination problem. In particular, the features of high order is more obvious during the aircraft through tactical maneuvers. In addition, due to the imprecision measurement and interactions with complex environments, networked nonlinear systems with uncertainties and external disturbances have to be investigated simultaneously in practice. For those complex nonlinear dynamics, the Brunovsky canonical form can been obtained through the linearization method. In the process of model transformation, the unmodeled dynamics and disturbances are embodied in the smooth nonlinear function and external disturbances.

Take the single-link flexible joint manipulator as a representative example, whose dynamics can be written as [15]

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -\frac{MgL}{I}\sin x_1 - \frac{k}{I}(x_1 - x_3),$$
$$\dot{x}_3 = x_4,\ \dot{x}_4 = \frac{k}{J}(x_1 - x_3) + \frac{1}{J}u,$$

where I, J are, respectively, the link and the rotor inertia moments, M is the link mass, k is the joint elastic constant, L is the distance from the axis of the rotation to the link center of mass and g is the gravitational acceleration, respectively. This nonlinear dynamics can be transformed to the higher-order normal form with $z_1 = x_1$ as

$$\dot{z}_1 = z_2,\ \dot{z}_2 = z_3,\ \dot{z}_3 = z_4,\ \dot{z}_4 = a(z) + b(z)u,$$

In this paper, we consider a group of m ($m \geq 2$) systems with non-identical dynamics distributed on an undirected communication network $\mathcal{G}$. The dynamics of the j-th system is described in the nonlinear Brunovsky form as

$$\dot{x}_{ij} = x_{(i+1),j} \tag{1}$$

$$\dot{x}_{nj} = u_j + f_j(x_j) + \zeta_j(t) \tag{2}$$

for $i = 1, \ldots, n-1$, where $x_{ij} \in \mathbf{R}$ is the i-th state of the j-th system; $x_j = [x_{1j}, \ldots, x_{nj}]^T \in \mathbf{R}^n$ is the state vector of system j; $f_j(x_j) : \mathbf{R}^n \to \mathbf{R}$ is locally Lipschitz with $f_j(0) = 0$, and it is assumed to be unknown; $u_j \in \mathbf{R}$ is the control protocol of the j-th system; $\zeta_j(t) \in \mathbf{R}$ is an external disturbance, including the noise, unstructured unmodeled dynamics, and approximation error from linearization, which is unknown but bounded. It's worth noting that this linearization is more empirical to easily use a priori knowledge to approximate the nonlinear function $f(x)$.

Assumption 1 The external disturbances $\zeta_j(t)$ for $j = 1, \ldots, m$ are unknown and bounded, that is, $|\zeta_j(t)| \leq \overline{\zeta}_j$ with $\overline{\zeta}_j$ being a known constant.

This bound $\overline{\zeta}_j$ is used to determine the parameters of the controller. In practice, the exact bound value is not the necessary condition, and any number larger than the precise bound value can be used for controller design.

Assumption 2 The communication graph $\mathcal{G}$ is fixed and connected.

The main goal of this paper is to design a distributed control law for the j-th system based on its own local states information when the communication topology is fixed and connected, such that

$$|x_{1j} - x_{1l}| \to 0, \text{ as } t \to \infty \text{ for } j, l = 1, \ldots, m. \tag{3a}$$

$$x_{ij} \to 0, \text{ as } t \to \infty \text{ for } i = 2, \ldots, n. \tag{3b}$$

2.3 Fuzzy logic systems on graph [26–29]

Since the nonlinear term $f_j(x_j)$ of the system (1–2) is unknown but with priori knowledge, in this paper, based on the fuzzy logic systems (FLS), the unknown function $f_j(x_j)$ can be approximated by $\hat{f}_j(x_j)$, where $\hat{f}_j(x_j) = \theta_j^T \phi_j(x)$, and $\phi_j(x_j) = [\phi_{1j}(x_j), \ldots, \phi_{nj}(x_j)]^T$ is a regressive vector. The knowledge base for the j-th FLS can be divided into some fuzzy IF-THEN rules and a fuzzy inference engine. By using product inference, center-average, and singleton fuzzifier [29], the output of the j-th FLS on graph can be expressed as

$$y_j(x_j) = \frac{\sum_{l=1}^{N} \overline{y}_{lj} \prod_{i=1}^{n} \mu_{F_{ij}^l}(x_{ij})}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_{ij}^l}(x_{ij})\right]} \tag{4}$$

where $x_j = (x_{1j}, \ldots, x_{nj})^T$ and y_j are the FLS input and output, respectively; $\overline{y}_{lj} = \max_{y_j \in R} \mu_{G_j^l}(y_j)$; F_{ij}^l and G_j^l are the fuzzy sets associating with the fuzzy functions $\mu_{F_{ij}^l}(x_{ij})$ and $\mu_{G_j^l}(y_j)$; N is the rule number of IF-THEN.

Define the fuzzy basis functions for the j-th system as

$$\phi_{lj} = \frac{\prod_{i=1}^{n} \mu_{F_{ij}^l}(x_{ij})}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_{ij}^l}(x_{ij})\right]} \tag{5}$$

Denoting $\theta_j^T = [\overline{y}_{1j}, \ldots, \overline{y}_{Nj}] = [\theta_{1j}, \ldots, \theta_{Nj}]$ and $\phi_j(x_j) = [\phi_{1j}(x_j), \ldots, \phi_{Nj}(x_j)]$, then FLS (4) can be rewritten as

$$y_j(x_j) = \theta_j^T \phi_j(x_j) \tag{6}$$

Lemma 1 [29]*: Let $f_j(x_j)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon_j > 0$, there exists an FLS* (6) *such that*

$$\sup_{x_j \in \Omega} |f_j(x_j) - \theta_j^T \phi_j(x_j)| \leq \varepsilon_j \tag{7}$$

3 Distributed control laws design

In this section, we show how to design distributed adaptive fuzzy controller for agents based on distributed backstepping techniques. Basic definitions are given and the local neighborhood virtual controllers are introduced. Some requirements on the topology are laid out, and a series of Lyapunov functions are given.

3.1 Local neighborhood virtual controllers

The systems (1)–(2) are with strict-feedback form. Owing to the structure character of the lower-triangular strict-feedback system, the high-rank state of each differential equation is used as virtual control. In this way, the consensus control problem for the higher-order multiple systems can be broken into a sequence of design problems for multiple lower-order subsystems. We need to define a set of new variables for virtual control design in distributed manner.

Definition 1 We define a set of new variables $z_{*j} = [z_{1j}, z_{2j}, \ldots, z_{nj}]^T$ with the aid of backstepping technique as follows

$$z_{1j} = x_{1j} \tag{8}$$

$$z_{ij} = x_{ij} - \alpha_{ij}, 2 \leq i \leq n \tag{9}$$

where $j = 1, \ldots, m$, α_{ij} is the virtual control function which is to be elaborately designed through recursive backstepping method. Both z_{*j} and α_{ij} are local.

3.2 Recursive controller design procedure

Different from the centralized scheme, the distributed recursive backstepping method only use the local information. The sequence of virtual controllers are also designed relying on local information. The highlight of this paper is to design the virtual controllers not only in a recursive way like other ordinary backstepping methods do, but also in a distributed manner which make the design procedure much more difficult than the centralized methods. Furthermore, due to the complex intrinsical nonlinearity described in formulation (2), an adaptive, fuzzy and robust control scheme must be considered synthetically. The actual controllers u_j can be derived from α_{nj} after α_{nj} is thoughtfully designed. The detailed recursive design procedure is given as follows:

In the first step, α_{2j} is used to denote the first virtual controller of system j. Using (1) for (8), it can be derived that

$$\dot{z}_{1j} = z_{2j} + \alpha_{2j} \tag{10}$$

Consider the error variable $z_{1j} = x_{1j}$ of the first-order subsystem of (1)–(2), and choose the Lyapunov function candidate V_1 as follows

$$V_1 = \frac{1}{2} z_{1*}^T z_{1*} \tag{11}$$

where $z_{1*} = [z_{11}, z_{12}, \ldots, z_{1m}]^T$.

Taking the time derivative of V_1 and following (9) and (10), we can obtain

$$\dot{V}_1 = \sum_{j=1}^{m} z_{1j}(z_{2j} + \alpha_{2j}) \tag{12}$$

Design the first distributed virtual controller α_{2j} as

$$\alpha_{2j} = -\sum_{l \in \mathcal{N}_j} a_{jl}(z_{1j} - z_{1l}) \tag{13}$$

where $\mathcal{N}_j$ denotes the neighbor set of the j-th agent and no global information states are included in α_{2j}, a_{jl} represents the weighted adjacency between the neighboring agents and all the a_{jl} are assumed to be 1 hereinafter. Information in communication networks only travels directly between immediate neighbors in a graph. Nevertheless, if a graph is connected, then this locally transmitted information travels ultimately to every agent in the graph.

With the aid of Eq.(13), (10) can be written as

$$\dot{z}_{1j} = -\sum_{l \in \mathcal{N}_j} (z_{1j} - z_{1l}) + z_{2j} \tag{14}$$

and $\dot{V}_1$ can be written as

$$\dot{V}_1 = -z_{1*}^T L z_{1*} + \sum_{j=1}^{m} z_{1j} z_{2j} \tag{15}$$

In the second step, by considering Eq.(9) and the second order of Eq.(1), it can be obtained that

$$\begin{aligned} \dot{z}_{2j} &= x_{3j} - \dot{\alpha}_{2j} \\ &= z_{3j} + \alpha_{3j} - \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} - \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \end{aligned} \tag{16}$$

where α_{3j} is treated as a virtual controller for a high-order subsystem which would be designed to guarantee the consensus of the first-order and the second-order subsystems for the multiple high-order systems. That is, the virtual controller α_{3j} is to be designed such that $\lim_{t \to \infty} (z_{1j} - z_{1l}) = 0$ and $\lim_{t \to \infty} z_{2j} = 0$ for $1 \le j \le m$.

Hence, choose the second Lyapunov function candidate V_2 as

$$V_2 = V_1 + \frac{1}{2} z_{2*}^T z_{2*} \tag{17}$$

where $z_{2*} = [z_{21}, z_{22}, \ldots, z_{2m}]^T$. Taking the time derivative of V_2 with respect to (15) and (16), we can get

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \sum_{j=1}^{m} z_{2j} \dot{z}_{2j} \\ &= -z_{1*}^T L z_{1*} + \sum_{j=1}^{m} z_{1j} z_{2j} + \sum_{j=1}^{m} z_{2j} \Bigg[z_{3j} + \alpha_{3j} \\ &\quad - \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} - \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \Bigg] \end{aligned} \tag{18}$$

In order to ensure that the time derivative of Lyapunov function V_2 is negative definite, an appropriate distributed virtual control α_{3j} should be designed. α_{3j} is designed as

$$\alpha_{3j} = -z_{1j} - c_{2j} z_{2j} + \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} + \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \tag{19}$$

where c_{2j} is the design parameter, satisfying $c_{2j} > 0$.

Note that α_{3j} only contains its own state information and neighbors' information without using any global information generally. The two items $-\frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j}$ and $-\sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l}$ in Eq. (18) are directly canceled by the design of α_{3j}. Furthermore, the item $-z_{1j}$ in α_{3j} is designed to make sure that the item $\sum_{j=1}^{m} z_{1j}z_{2j}$ in Eq. (18) can be eliminated. And the item $-c_{2j}z_{2j}$ in Eq. (19) is designed to ensure the negative definite of the Eq. (18). The item $\sum_{j=1}^{m} z_{2j}z_{3j}$ in Eq. (20) will be handled in the third step by choosing an appropriate virtual controller α_{4j}.

Therefore, by substituting (19) into (18), $\dot{V}_2$ can be rewritten as follows

$$\dot{V}_2 = -z_{1*}^T L z_{1*} - z_{2*}^T \mathrm{diag}(c_{2*}) z_{2*} + \sum_{j=1}^{m} z_{2j} z_{3j} \tag{20}$$

where $c_{2*} = [c_{21}, c_{22}, \ldots, c_{2m}]^T$.

In step i, where $1 \le i \le n-1$. Follow the design procedure which is similar to the first and second step, it can be obtained that

$$\begin{aligned} \dot{z}_{ij} &= x_{(i+1)j} - \dot{\alpha}_{ij} \\ &= z_{(i+1)j} + \alpha_{(i+1)j} - \sum_{k=1}^{i-1} \frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad - \sum_{k=1}^{i-1} \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l} \end{aligned} \tag{21}$$

In (21), the virtual controller $\alpha_{(i+1)j}$ which can guarantee the consensus of the multiple i-rank $(1 < i < n-1)$ subsystems would be designed such that $\lim_{t\to\infty} z_{kj} = 0$ for $1 \le j \le m$ and $1 \le k \le n-1$, with the aid of the Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2} z_{i*}^T z_{i*} \tag{22}$$

Note that V_{i-1} can be designed in the $i-1$ step by the recursive method. Taking the time derivative of V_i with considering V_{i-1} in step i and (21), we can get

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} + \sum_{j=1}^{m} z_{ij}\dot{z}_{ij} \\ &= -z_{1*}^T L z_{1*} - \sum_{j=2}^{i-1} z_{j*}^T \mathrm{diag}(c_{j*}) z_{j*} \\ &\quad + \sum_{j=1}^{m} z_{(i-1)j} z_{ij} + \sum_{j=1}^{m} z_{ij} \Bigg[-\sum_{k=1}^{i-1} \frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad + z_{(i+1)j} + \alpha_{(i+1)j} - \sum_{k=1}^{i-1} \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l} \Bigg] \end{aligned} \tag{23}$$

Choose the virtual controller $\alpha_{(i+1)j}$ as

$$\begin{aligned} \alpha_{(i+1)j} &= -z_{(i-1)j} - c_{ij} z_{ij} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad + \sum_{k=1}^{i-1} \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l} \end{aligned} \tag{24}$$

where c_{ij} is the design parameter and satisfy $c_{ij} > 0$. Substituting (24) into $\dot{V}_i$, it is obtained that

$$\begin{aligned} \dot{V}_i &= -z_{1*}^T L z_{1*} - \sum_{j=2}^{i} z_{j*}^T \mathrm{diag}(c_{j*}) z_{j*} \\ &\quad + \sum_{j=1}^{m} z_{ij} z_{(i+1)j} \end{aligned} \tag{25}$$

where $c_{i*} = [c_{i1}, c_{i2}, \ldots, c_{im}]^T$.

In the last step, FLS is used to approximate the unknown term $f_j(x_j)$ of the nonlinear dynamics (2). Define the minimal approximation error $\varepsilon_j = f_j(x_j) - f_j(x_j|\theta_j^*)$, where $f_j(x_j|\theta_j^*) = \theta_j^{*T}\phi_j(x_j)$, and θ_j^* is the optimal fuzzy parameter vector. Based on the FLS (4)–(6), the unknown function $f_j(x_j)$ can be approximated by $\hat{f}_j(x_j) = \hat{\theta}_j^T \phi_j(x_j)$, where $\hat{\theta}_j$ is the estimation of θ_j^*, and $\phi_j(x_j) = [\phi_{1j}(x_j), \ldots, \phi_{nj}(x_j)]^T$ is a regressive vector.

Assumption 3 [33,34] There exists a known positive constant $\bar{\varepsilon}_j$, such that $|\varepsilon_j| \le \bar{\varepsilon}_j$.

Remark 1 By Lemma 1, a fuzzy logic system has the approximation capability for any continuous smooth function. Thus, it is generally assumed that the fuzzy minimum approximation errors ε_j $(j = 1, \ldots, m)$ are bounded with known upper bounds $\bar{\varepsilon}_j$, for example, [33,34] and the references therein. In this subsection, Assumption 3 is adopted. However, in practice, it is difficult to determine the upper bounds $\bar{\varepsilon}_j$. In literatures, the approach to estimating them online via adaptation laws is proposed [35–37]. In Sect. 3.3, we will discuss the design procedure in detail when the parameter $\bar{\epsilon}_j$ is unknown.

By applying the results in the 1 to $n-1$ steps, it can be obtained that

$$\begin{aligned} \dot{z}_{nj} &= u_j + f_j(x_j) + \zeta_j - \dot{\alpha}_{nj} \\ &= -\sum_{k=1}^{n-1} \frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1),j} - \sum_{k=1}^{n-1} \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1),l} \\ &\quad + f_j(x_j|\theta_j^*) + \varepsilon_j + \zeta_j + u_j \end{aligned} \tag{26}$$

Hence, choose the n-th Lyapunov function candidate V_n as

$$V_n = V_{n-1} + \frac{1}{2} z_{n*}^T z_{n*} + \frac{1}{2} \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \tilde{\theta}_j \tag{27}$$

where $\tilde{\theta}_j = \theta_j^* - \hat{\theta}_j$ is the fuzzy parameter error vector.

Taking the time derivative of V_n with respect to (25) and (26), we obtain

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \sum_{j=1}^{m} z_{nj}\dot{z}_{nj} + \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \dot{\tilde{\theta}}_j \\ &= -z_{1*}^T L z_{1*} - \sum_{i=2}^{n-1} z_{i*}^T \text{diag}(c_{i*}) z_{i*} + \sum_{j=1}^{m} z_{(n-1)j} z_{nj} \\ &\quad + \sum_{j=1}^{m} z_{nj} \left[u_j + f_j(x_j|\theta_j^*) + \varepsilon_j - \sum_{k=1}^{n-1} \frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1)j} \right. \\ &\quad \left. - \sum_{k=1}^{n-1} \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1)l} + \zeta_j \right] + \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \dot{\tilde{\theta}}_j\end{aligned} \tag{28}$$

We choose the adaptation laws:

$$\dot{\hat{\theta}}_j = \Gamma_j z_{nj} \phi_j \tag{29}$$

where Γ_j is positive definite matrices, Note that z_{nj} only contains the local information. And the distributed control law is

$$\begin{aligned}u_j &= -z_{(n-1)j} - c_{nj} z_{nj} + \sum_{k=1}^{n-1} \frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad + \sum_{k=1}^{n-1} \sum_{l \in \mathcal{N}_j} \frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1)l} - \hat{\theta}_j^T \phi_j(x_j) \\ &\quad - \overline{\epsilon}_j \text{sign}(z_{nj})\end{aligned} \tag{30}$$

where $-\overline{\epsilon}_j \text{sign}(z_{nj})$ is the robustness term, which is utilized to eliminate the effect of the FLC approximation error and external disturbances, and $\overline{\epsilon}_j \geq \overline{\varepsilon}_j + \overline{\zeta}_j$.

Using (29) and (30) for (28), we can derive

$$\begin{aligned}\dot{V}_n &= -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \text{diag}(c_{i*}) z_{i*} \\ &\quad + \sum_{j=1}^{m} z_{nj} [f_j(x_j|\theta_j^*) - \hat{f}_j(x_j) - \overline{\epsilon}_j \text{sign}(z_{nj}) \\ &\quad + \varepsilon_j + \zeta_j] + \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \dot{\tilde{\theta}}_j \\ &= -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \text{diag}(c_{i*}) z_{i*} \\ &\quad + \sum_{j=1}^{m} z_{nj} [\theta_j^{*T} \phi_j(x_j) - \hat{\theta}_j^T \phi_j(x_j) - \overline{\epsilon}_j \text{sign}(z_{nj}) \\ &\quad + \varepsilon_j + \zeta_j] + \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \dot{\tilde{\theta}}_j \\ &\leq -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \text{diag}(c_{i*}) z_{i*} \\ &\quad + \sum_{j=1}^{m} z_{nj} (\varepsilon_j + \zeta_j) \\ &\quad + \sum_{j=1}^{m} z_{nj} \tilde{\theta}_j^T \phi_j(x_j) + \sum_{j=1}^{m} \tilde{\theta}_j^T \Gamma_j^{-1} \dot{\tilde{\theta}}_j \\ &\quad - \sum_{j=1}^{m} \overline{\epsilon}_j |z_{nj}| \\ &\leq -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \text{diag}(c_{i*}) z_{i*} \\ &\quad + \sum_{j=1}^{m} (\overline{\varepsilon}_j + \overline{\zeta}_j) |z_{nj}| - \sum_{j=1}^{m} \overline{\epsilon}_j |z_{nj}| \leq 0\end{aligned} \tag{31}$$

Theorem 1 *Consider the multiple nonlinear systems described by* (1)–(2)*, when the Assumptions* 1–3 *are satisfied, choose the control law* (30) *and the adaptation law* (29) *for system j, where $1 \leq j \leq m$, then it guarantees that the control objective* (3) *holds, that is the consensus of high-order nonlinear uncertain systems can be reached asymptotically.*

Proof By the above design procedure, define the Lyapunov function candidate as (27), then we get (31). Therefore, it follows that $z_{i*} \in \mathcal{L}^\infty$, $\tilde{\theta}_j \in \mathcal{L}^\infty$ and $\hat{\theta}_j$ is bounded according to the boundedness of θ_j. From (8), (9), (13), we get x_{1j}, α_{2j} and x_{2j} are bounded, furthermore, α_{3j} is bounded from (19), following this procedure, we claim that u_j is bounded. Using the above arguments, it follows that $\dot{z}_{i*}, \dot{\tilde{\theta}}_j$ are all bounded from (15), (18), (23), (28), (31) and the definition of ϕ_j and Γ_j. By differentiating (31), we can see that $\ddot{V}_n$ is bounded, which means $\dot{V}_n$ is uniformly continuous. Hence, using Barbalat's lemma [20], it follows that $\dot{V}_n \to 0$ as $t \to \infty$, i.e., $\lim_{t\to\infty} z_{1*}^T L z_{1*} = 0$ and $\lim_{t\to\infty} z_{l*} = \mathbf{0}_m$ for $2 \leq l \leq n$. Using $\lim_{t\to\infty} z_{2*} = \mathbf{0}_m$, (10) becomes $\dot{z}_{1j} = -\sum_{l\in\mathcal{N}_j} a_{jl}(z_{1j} - z_{1l})$, which implies that $\dot{x}_{1j} = -\sum_{l\in\mathcal{N}_j} a_{jl}(x_{1j} - x_{1l})$, thus the consensus is reached by Lemma 2.10 in [16], i.e., for all $x_{1j}(0)$ and all $i, j = 1, \ldots, m$, $|x_{1i} - x_{1j}| \to 0$ as $t \to \infty$. We then obtain $\lim_{t\to\infty} Lx_{1*} = \mathbf{0}_m$ and $\lim_{t\to\infty} x_{1*} = a\mathbf{1}_m$ for some $a \in \mathbf{R}$ according to $L\mathbf{1}_m = \mathbf{0}_m$, where $\mathbf{1}_m$ and $\mathbf{0}_m$ denote the $m \times 1$ column vector of all ones and zeros. Now, denote $\bar{x}_1 = \frac{1}{n}\sum_{j=1}^m x_{1j}$ as the average of the first-order states, we get $\dot{\bar{x}}_1 = \frac{1}{n}\sum_{j=1}^m \dot{x}_{1j} = -\frac{1}{n}\mathbf{1}_m^T Lx_{1*} = 0$, so that $\dot{\bar{x}}_1 = \frac{1}{n}\sum_{j=1}^m x_{1j}(0)$, which means $a = \frac{1}{n}\sum_{j=1}^m x_{1j}(0)$, and the average consensus of the first-order states has been reached. When $\lim_{t\to\infty}(x_{1i} - x_{1j}) = 0$, we get $\alpha_{2j} \to 0$ as $t \to \infty$, thus $x_{2j} \to 0$ will hold from (11) and (15). Following this step, we can further obtain that $x_{ij} \to 0$ as $t \to \infty$ for $i = 3, \ldots, n$.

Remark 2 The proposed control law contains the sign function, thus may lead to control chattering. This situation can be remedied by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface. To do this, the sign function in the control law (30) can be replaced by a saturation function

$$\mathrm{sat}(z_{nj}) = \begin{cases} 1 & \text{if } z_{nj}/\varphi_j \ge 1 \\ z_{nj}/\varphi_j & \text{if } -1 < z_{nj}/\varphi_j < 1 \\ -1 & \text{if } z_{nj}/\varphi_j \le -1 \end{cases}$$

where $\varphi_j > 0$ is the boundary layer thickness. Though the boundary layer leads to small terminal tracking errors, the advantages of practical use may be significant [38]. To reject the control chattering and tackle the problem discussed in *Remark 1*, a modified control structure will be developed in the following subsection.

3.3 Modified control design

In literatures, Assumptions 1 and 3 are commonly used. However, in this subsection, the condition that the bounds of the external disturbances and approximation error are unknown is considered. Thus the Assumptions 4 and 5 will be adopted as follows.

Assumption 4 The unknown external disturbances $\zeta_j(t)$ for $j = 1, \ldots, m$ are bounded with $|\zeta_j(t)| \le \overline{\zeta}_j$, where $\overline{\zeta}_j$ is an unknown constant.

Assumption 5 [35–37] There exist unknown positive constant $\bar{\varepsilon}_j$, such that $|\varepsilon_j| \le \bar{\varepsilon}_j$.

Based on Assumptions 4 and 5, we will discuss the situation when $\bar{\epsilon}_j$ is unknown in this subsection. An adaptive parameter $\hat{\epsilon}_j$ will be used to estimate $\bar{\epsilon}_j$, where $\bar{\epsilon}_j \ge \bar{\varepsilon}_j + \overline{\zeta}_j$. The detailed design process is given in the following analysis.

Denote $z'_{nj} = z_{nj} - \varphi_j \mathrm{sat}(z_{nj})$, where $\varphi_j > 0$. The modify control law and adaptive laws are proposed as follows:

$$\dot{\hat{\theta}}_j = \Gamma_j z'_{nj}\phi_j(x_j) \tag{32}$$

$$\dot{\hat{\epsilon}}_j = \kappa_{1j}|z'_{nj}| \tag{33}$$

$$\begin{aligned} u_j = & -z_{(n-1)j} - c_{nj}z'_{nj} + \sum_{k=1}^{n-1}\frac{\partial\alpha'_{nj}}{\partial x_{kj}}x_{(k+1)j} \\ & + \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha'_{nj}}{\partial x_{kl}}x_{(k+1)l} - \hat{\theta}_j^T\phi_j(x_j) \\ & - \hat{\epsilon}_j\mathrm{sat}(z_{nj}) \end{aligned} \tag{34}$$

$$\begin{aligned} \alpha'_{nj} = & -z_{(n-2)j} - c_{(n-1)j}z_{(n-1)j} - \varphi_j\mathrm{sign}(z_{(n-1)j}) \\ & + \sum_{k=1}^{i-1}\frac{\partial\alpha_{ij}}{\partial x_{kj}}x_{(k+1)j} + \sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{ij}}{\partial x_{kl}}x_{(k+1)l} \end{aligned} \tag{35}$$

where κ_{1j} is a positive design parameters.

Theorem 2 *Consider the multiple nonlinear systems described by* (1)–(2)*, when the Assumptions* 2*,* 4 *and* 5 *are satisfied, choose the control law* (34) *and the adaptation laws* (32)–(33) *for system* j*, where* $1 \le j \le m$*, then it guarantees that the objectives* (3a), (3a) *for* $i = 2, \ldots, n-1$ *and* $|x_{nj}| < \varphi_j$ as $t \to \infty$ *hold.*

Proof The proving procedures are the same as theorem 1 in step 1∼ $(n-2)$.

In step $(n-1)$, α_{nj} is redesigned as α'_{nj} in (35). The Lyapunov function candidate V_{n-1} is defined as $V_{n-1} = V_{n-2} + \frac{1}{2}z_{(n-1)*}^T z_{(n-1)*}$. Taking the time derivative of V_{n-1}, we get

$$\begin{aligned} \dot{V}_{n-1} = & -z_{1*}^T Lz_{1*} - \sum_{j=2}^{n-2} z_{j*}^T \mathrm{diag}(c_{j*})z_{j*} \\ & + \sum_{j=1}^{m} z_{(n-2)j}z_{(n-1)j} \end{aligned}$$

$$+\sum_{j=1}^{m} z_{(n-1)j}\left[-\sum_{k=1}^{n-2}\frac{\partial\alpha_{(n-1)j}}{\partial x_{kj}}x_{(k+1)j}\right.$$
$$\left.+z_{nj}+\alpha'_{nj}-\sum_{k=1}^{n-2}\sum_{l\in\mathcal{N}_j}\frac{\partial\alpha_{(n-1)j}}{\partial x_{kl}}x_{(k+1)l}\right]$$

Substituting (35) into $\dot{V}_{n-1}$, we obtain

$$\dot{V}_{n-1}=-z_{1*}^T L z_{1*}-\sum_{j=2}^{n-1} z_{j*}^T \mathrm{diag}(c_{j*})z_{j*}$$
$$+\sum_{j=1}^{m} z_{(n-1)j}z_{nj}-\sum_{j=1}^{m}\varphi_j|z_{(n-1)j}| \tag{36}$$

In step n, we define the Lyapunov function candidate V'_n as

$$V'_n=V_{n-1}+\frac{1}{2}z'^T_{n*}z'_{n*}+\frac{1}{2}\sum_{j=1}^{m}\tilde{\theta}_j^T\Gamma_j^{-1}\tilde{\theta}_j$$
$$+\frac{1}{2}\sum_{j=1}^{m}\kappa_{1j}^{-1}\tilde{\epsilon}_j^2 \tag{37}$$

where $z'_{n*}=[z'_{n1},z'_{n2},\ldots,z'_{nm}]^T$, $\tilde{\epsilon}_j=\bar{\epsilon}_j-\hat{\epsilon}_j$. Using the fact that $|z'_{nj}|=0$ for $|z_{nj}|<\varphi_j$ and $|z'_{nj}|=z'_{nj}\mathrm{sat}(z_{nj})$ for $|z_{nj}|\ge\varphi_j$, $\dot{z}'_{nj}=\dot{z}_{nj}$, and Eqs. (32)–(34), it results in

$$\dot{V}'_n=-z_{1*}^T L z_{1*}-\sum_{i=2}^{n-1} z_{i*}^T \mathrm{diag}(c_{i*})z_{i*}-\sum_{j=1}^{m}\varphi_j|z_{(n-1)j}|$$
$$-z'^T_{n*}\mathrm{diag}(c_{n*})z'_{n*}+\sum_{j=1}^{m}\varphi_j z_{(n-1)j}\mathrm{sat}(z_{nj})$$
$$+\sum_{j=1}^{m} z'_{nj}[\theta_j^{*T}\phi_j(x_j)-\hat{\theta}_j^T\phi_j(x_j)-\hat{\epsilon}_j\mathrm{sat}(z_{nj})$$
$$+\varepsilon_j+\zeta_j]+\sum_{j=1}^{m}\tilde{\theta}_j^T\Gamma_j^{-1}\dot{\tilde{\theta}}_j+\sum_{j=1}^{m}\kappa_{1j}^{-1}\tilde{\epsilon}_j\dot{\tilde{\epsilon}}_j$$
$$\le -z_{1*}^T L z_{1*}-\sum_{i=2}^{n-1} z_{i*}^T \mathrm{diag}(c_{i*})z_{i*}-\sum_{j=1}^{m}\varphi_j|z_{(n-1)j}|$$
$$-z'^T_{n*}\mathrm{diag}(c_{n*})z'_{n*}+\sum_{j=1}^{m}\varphi_j|z_{(n-1)j}|$$
$$+\sum_{j=1}^{m} z'_{nj}\tilde{\theta}_j^T\phi_j(x_j)+\sum_{j=1}^{m}\tilde{\epsilon}_j|z'_{nj}|$$
$$+\sum_{j=1}^{m}\tilde{\theta}_j^T\Gamma_j^{-1}\dot{\tilde{\theta}}_j+\sum_{j=1}^{m}\kappa_{1j}^{-1}\tilde{\epsilon}_j\dot{\tilde{\epsilon}}_j \tag{38}$$

Substituting (32)–(33) into (38), it yields

$$\dot{V}'_n\le -z_{1*}^T L z_{1*}-\sum_{i=2}^{n-1} z_{i*}^T \mathrm{diag}(c_{i*})z_{i*}$$
$$-z'^T_{n*}\mathrm{diag}(c_{n*})z'_{n*}\le 0 \tag{39}$$

Similar to theorem 1, from (39), it follows that $z_{i*}\in\mathcal{L}^\infty$ for $i=1,\ldots,n-1$, $z'_{n*}\in\mathcal{L}^\infty$, $\tilde{\theta}_j\in\mathcal{L}^\infty$, $\tilde{\epsilon}_j\in\mathcal{L}^\infty$ and $\hat{\theta}_j$, $\hat{\epsilon}_j$ are bounded according to the boundedness of θ_j^* and $\bar{\epsilon}_j$. From (8), (9), (13), we get x_{1j}, α_{2j} and x_{2j} are bounded, furthermore, α_{3j} is bounded from (19), following this procedure, we claim that u_j is bounded. Using the above arguments, it follows that $\dot{z}_{i*}$ for $i=1,\ldots,n-1$, $\dot{z}'_{n*}$, $\dot{\tilde{\theta}}_j$ and $\dot{\tilde{\epsilon}}_j$ are all bounded from (15), (18), (23), (36), (39) and the definition of ϕ_j, Γ_j and κ_{1j}. By differentiating (39), we can see that $\ddot{V}'_n$ is bounded, which means $\dot{V}'_n$ is uniformly continuous. Hence, using Barbalat's lemma [20], it follows that $\dot{V}'_n\to 0$ as $t\to\infty$, i.e., $\lim_{t\to\infty} z_{1*}^T L z_{1*}=0$, $\lim_{t\to\infty} z_{l*}=\mathbf{0}_m$ for $2\le l\le n-1$ and $\lim_{t\to\infty} z'_{n*}=\mathbf{0}_m$. Following the analysis of Theorem 1, we can easily get $|x_{1j}-x_{1l}|\to 0$ as $t\to\infty$ for $j,l=1,\ldots,m$, and $x_{ij}\to 0$ as $t\to\infty$ for $i=2,\ldots,n-1$. Then we can further get $|x_{nj}|<\varphi_j$ as $t\to\infty$ for $i=2,\ldots,n-1$ using the fact that $z'_{nj}\to 0$ and $\alpha'_{nj}\to 0$ as $t\to\infty$. This completes the proof.

Remark 3 By considering the structure characteristics of the system, the key idea of our proposed method is to break a huge consensus problem with the multiple high-order nonlinear systems into a sequence of recursive design problems with lower-order multiple subsystems based on the backstepping frameworks. In each step of the design procedure, only local information is used to design the virtual controller, which makes it more difficult to find the appropriate controllers, but consequently the resulting actual controller and the parameter adaptive law can be obtained in a distributed manner, which overcomes the main drawbacks of the ordinary backstepping methods in which global state information must be used.

Remark 4 As we know, on the study of nonlinear function approximation, fuzzy control, and neural networks have many similarities. These two technologies can both approximate any real continuous functions on a compact set. However, with respect the complex n-order nonlinear systems whose unknown dynamics has valuable priori knowledge, especially, derived from linearization and reduced-order approximation, fuzzy control has important merits that it can fully utilize the priori knowledge. The training of fuzzy control is done by the commonsense rules library. In contrast, the neural network control need to be trained, so it lead to larger amount of calculation.

4 Simulation

In this section, due to the limitation of space, we only give an example for Theorem 2 which is used to show the effectiveness of the proposed distributed adaptive fuzzy control law (34). Consider a five node undirected graph described in Fig. 1. Note that the communication graph $\mathcal{G}$ satisfies Assumption 2. For simplicity, the corresponding adjacent weights between the networked systems are assumed to be 1, and all the others are 0.

Consider the following four-order uncertain nonlinear dynamics:

$$\dot{x}_{1j} = x_{2j}$$
$$\dot{x}_{2j} = x_{3j}$$
$$\dot{x}_{3j} = x_{4j}$$
$$\dot{x}_{4j} = u_j + f_j(x_{1j}, x_{2j}, x_{3j}, x_{4j}) + \zeta_j(t)$$

with

$$\dot{x}_{41} = u_1 + 0.2(x_{11} + x_{41}) + 0.3\sin\left(\frac{t}{5}\right) + 6\cos(6t)$$
$$\dot{x}_{42} = u_2 + (x_{12} + x_{22} - 1)^2 + 0.3\sin\left(\frac{t}{5}\right) + 3\sin(2t)$$
$$\dot{x}_{43} = u_3 + 0.3\cos(x_{13} + x_{23}) + 0.3\sin\left(\frac{t}{5}\right) + 1$$
$$\dot{x}_{44} = u_4 + 0.2\sin(x_{14} + x_{24}) + \cos(3t) - \sin(t) + 0.2$$
$$\dot{x}_{45} = u_5 + 0.2\sin(x_{15}) + \cos(2t)$$

The initial state information and the disturbances of the systems are:

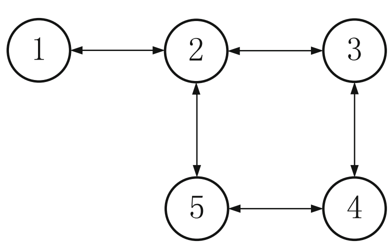

Fig. 1 Communication graph $\mathcal{G}$ of the multiple nonlinear systems

$x_{1j}(0) = [1, 0.3, 1, -0.5]^T$, $x_{2j}(0) = [-0.5, 1, 1, -1]^T$, $x_{3j}(0) = [1.5, -1, -0.2, 3]^T$, $x_{4j}(0) = [-0.2, -1, 0.1, 1]^T$, $x_{5j}(0) = [-1.75, -0.2, 0.1, 0.2]$.

Define fuzzy membership as follows: $\mu_{F_4^l}(x_{1j}, x_{2j}, x_{3j}, x_{4j}) = \exp[-(x_{1j} - 3 + l)^2/2] \times \exp[-(x_{2j} - 3 + l)^2/2] \times \exp[-(x_{3j} - 3 + l)^2/2] \times \exp[-(x_{4j} - 3 + l)^2/2]$, $l = 1, \ldots, 5$. We obtain fuzzy basis functions as follows:

$$\phi_{4p}(x_{1j}, x_{2j}, x_{3j}, x_{4j}) = \frac{\exp\left[\frac{-(x_{1j}-3+p)^2}{2}\right] \times \ldots \times \exp\left[\frac{-(x_{4j}-3+p)^2}{2}\right]}{\sum_{n=1}^{5} \exp\left[\frac{-(x_{1j}-3+n)^2}{2}\right] \times \ldots \times \exp\left[\frac{-(x_{4j}-3+n)^2}{2}\right]}$$

where $p = 1, \ldots, 5$.

The FLSs can be expressed in the following form: $\hat{f}_j(x_j|\theta_j) = \hat{\theta}_j^T \phi_j(x_j)$, where $\hat{\theta}_j^T = [\hat{\theta}_{1j}, \hat{\theta}_{2j}, \hat{\theta}_{3j}, \hat{\theta}_{4j}, \hat{\theta}_{5j}]$, and $\phi_j(x_j) = [\phi_{1j}(x_j), \phi_{2j}(x_j), \phi_{3j}(x_j), \phi_{4j}(x_j), \phi_{5j}(x_j)]$.
with the initial state information:

$$\hat{\theta}_1(0) = [0.01, 0.02, 0.01, 0.01, 0.01]^T,$$
$$\hat{\theta}_2(0) = [0.1, -0.01, 0.02, 0.05, 0.02]^T,$$
$$\hat{\theta}_3(0) = [0.3, 0.2, -0.3, 0.4, 0.3]^T,$$
$$\hat{\theta}_4(0) = [-0.06, 0.03, 0.07, 0.1, -0.02]^T,$$
$$\hat{\theta}_5(0) = [0.3, 0.2, -0.3, 0.4, 0.3]^T.$$

$\hat{\epsilon}_1(0) = 7.1$, $\hat{\epsilon}_2(0) = 6.1$, $\hat{\epsilon}_3(0) = 6.1$, $\hat{\epsilon}_4(0) = 6.1$, $\hat{\epsilon}_5(0) = 15.1$, $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = \Gamma_5 = 5.2I$, $\kappa_{11} = \kappa_{12} = \kappa_{13} = \kappa_{14} = \kappa_{15} = 1$, $c_{ij} = 1$ for $i = 2, 3, 4$, $j = 1, \ldots, 5$. The thin boundary layer ϕ_j for $j = 1, \ldots, 5$ neighboring the switching surface is 0.05.

Figure 2 shows the control torque of each agent by the distributed control law in (31). Figures 3, 4, 5 and 6 shows the time histories of state trajectories for each agent. From Fig. 3, it can be seen that, under the control torque which are shown in Fig. 2, the consensus is achieved. These figures demonstrate the efficiency of the proposed algorithm in guaranteeing consensus despite the presence of complex unknown dynamics. Therefore, the distributed consensus control laws in Theorem 2 are effective.

5 Conclusion

This paper considered the cooperative consensus control problem of networked high-order nonlinear systems with distinct unknown dynamics and bounded

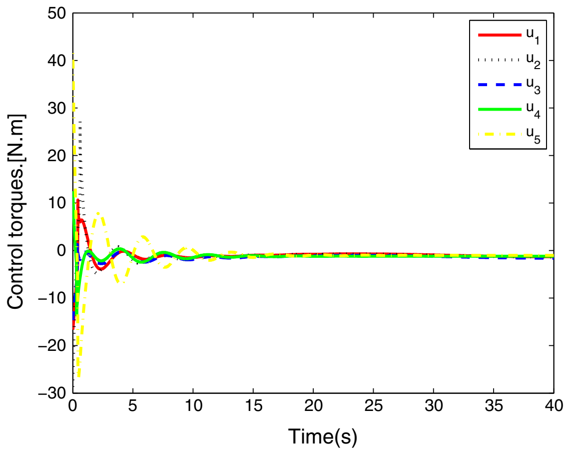

Fig. 2 Response of u_j for $1 \leq j \leq 5$

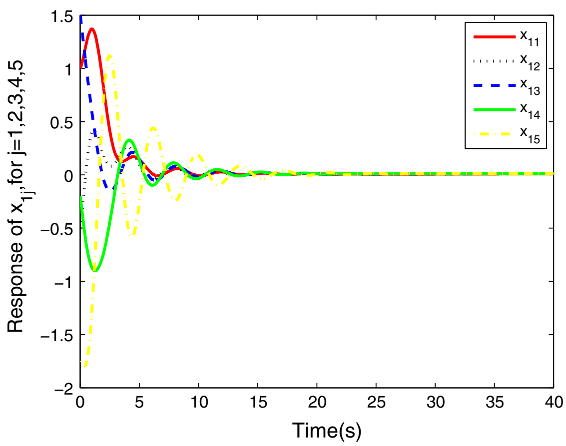

Fig. 3 Response of x_{1j} for $1 \leq j \leq 5$

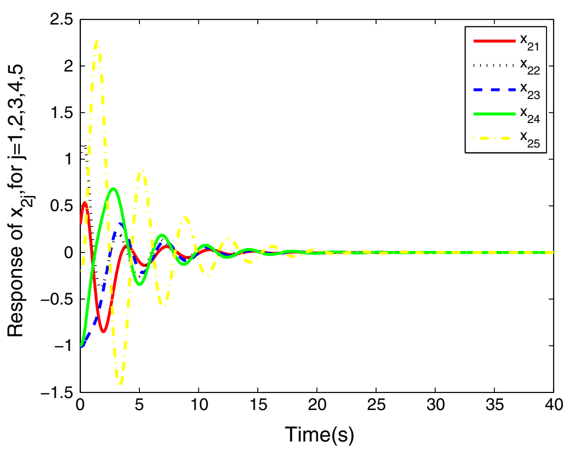

Fig. 4 Response of x_{2j} for $1 \leq j \leq 5$

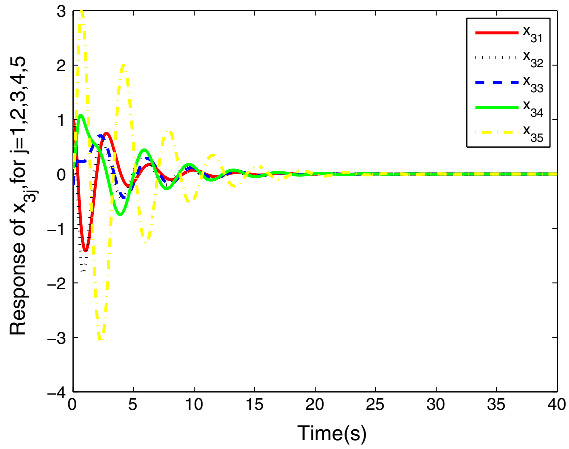

Fig. 5 Response of x_{3j} for $1 \leq j \leq 5$

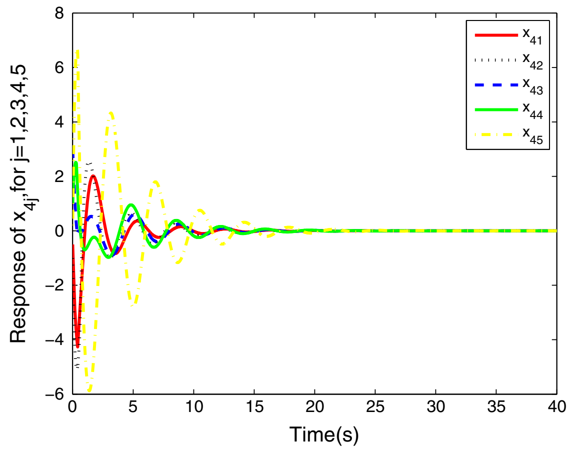

Fig. 6 Response of x_{4j} for $1 \leq j \leq 5$

external disturbances. The nonlinearities are only assumed to be locally Lipschitz. Two adaptive fuzzy control algorithms were proposed under the distributed backstepping framework. The proposed algorithms are completely distributed in the sense that the controller for each agent only uses information of itself and its neighbors. This backstepping control design is in fact distributed over a communication network, which be reflected in the virtual control and further be reflected in the final adaptive fuzzy control algorithm.

There are relevant problems that need to be investigated. For example, the topologies for the practical multi-agent networks may change over time. Consensus problems for multi-agent systems with switching/directed topologies are more complicated because the neighboring set of each agent is time varying.

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