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# Multiple-model adaptive robust dynamic surface control with estimator resetting

Minggang Gan, Jie Chen and Zhiping Li*,†

*School of Automation, Beijing Institute of Technology, Beijing 100081, China*

SUMMARY

A multiple-model adaptive robust dynamic surface control with estimator resetting is investigated for a class of semi-strict feedback nonlinear systems in this paper. The transient performance is mainly considered. The multiple models are composed of fixed models, one adaptive model, and one identification model that can be obtained when the persistent exciting condition is satisfied. The transient performance of the final tracking system can be improved significantly by designing proper switching mechanism during the parameter tuning procedure. The semi-globally uniformly ultimately bounded stability of the closed-loop system can be easily achieved because of the framework of adaptive robust dynamic surface control. Numerical examples are provided to demonstrate the effectiveness of the proposed multiple-model controller. Copyright © 2014 John Wiley & Sons, Ltd.

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KEY WORDS: adaptive robust control; multiple-model; dynamic surface control; transient performance

## 1. INTRODUCTION

The main factors that deteriorate the performance of control system may be the unknown parameters, unmodeled system dynamics, and external disturbances. Yao viewed these uncertainties as parametric and nonlinear uncertainties and proposed an adaptive robust control (ARC) framework, to solve these uncertainties efficiently [1]. In this framework, the advantages of adaptive and robust control are effectively combined, and the conflicts between these two control methods are dealt with by using the projection operator in a parameter adaptation law. As long as the parameter estimates stay in the prescribed set, the ARC can lead the system to a good performance because of its strong robust. Thus, it is widely used and effective in many real engineering applications, especially in servomechanisms [2–4].

However, in modern control engineering practices, accurate final tracking error with fast system response may be more popularized. In the normal ARC approach, prescribed transient performance will be realized by adjusting the robust control gain, and asymptotical convergence of tracking error can be achieved with long learning procedure only in the presence of parametric uncertainties. That is to say, better transient performance needs a higher control gain, which is very harmful to the stability of the system at the initial time. More unfortunately, the higher control gain cannot do any contribution to improving the asymptotical tracking performance because the adaptive control term does not do any contribution to accelerating the convergence speed of the unknown parameters. In the framework of ARC, the parameter adaptation law is usually derived from Lyapunov stability analysis to remove the crossed term. The transient performance was guaranteed by using high control and learning gains. In [5, 6], a composite adaptation law was proposed to accelerate

*Correspondence to: Zhiping Li, School of Automation, Beijing Institute of Technology, Beijing 100081, China.
†E-mail: lizplst@gmail.com

the adaptation speed. However, when the initial condition go against the parameter estimation, the transient problem still exists.

In the past 20 years, to enhance the transient performance of the control system, Morse and Narendra and Balakrishnan have proposed the multiple-model control concept in the adaptive control community for linear systems [7, 8]. It has been proved that the multiple-model adaptive control is a well-established approach for implementing adaptive systems with fast transient response. In [9], Narendra and Balakrishnan discussed different combination of multiple models including all adaptive models, all fixed models, fixed models and one adaptive model, and fixed models with one free-running and one reinitialized adaptive model. Subsequently, the multiple model approach was expanded to nonlinear systems [10]. For example, adaptive control using multiple identification models for a class of parametric strict feedback systems was investigated, where all adaptive models combination was adopted [11]. In [12, 13], nonlinear adaptive backstepping using estimator resetting based on multiple models was investigated, and a new switching and resetting criteria were proposed by monitoring the negative jump in the control Lyapunov function. In [14–16], Ciliz *et al*. did a lot of work in switching and resetting mechanisms and criteria for the multiple models adaptive control of nonlinear systems.

In this paper, we focus on the transient performance of adaptive robust dynamic surface control (ARDSC) for a class of semi-strict parametric feedback nonlinear systems. Multiple-model ARC with switching, tuning, and estimator resetting is considered. However, the fixed models are difficult to determine. Commonly, if more fixed models are selected, the accurate model will be approached more probably. Nevertheless, larger computation cost will arise. To solve this problem, in our previous work [17], an identifier-based ARC of servomechanism was designed to improve the tracking transient performance where only an identification model and an adaptive model are used. However, the transient performance is only depended on the persistent exciting (PE) condition. To overcome these problems, the multiple models, which consist of numbers of fixed models, an identification model, and a reinitialized adaptive model, are introduced in this paper. The fixed models are used as switching candidates to improve the system tracking transient performance till the identification model is constructed. When the PE condition is satisfied, an accurate fixed model can be obtained, which results in an easier determination of the fixed models. The reinitialized adaptive model occurs according to a common criterion, which is similar to the ones in [13–15]. A modular adaptive design philosophy will guarantee the stabilization of the closed-loop tracking system.

The remainder of this paper is organized as follows. In Section 2, problem formulation and preliminaries are provided. In Section 3, multiple models ARDSC with parameter resetting is investigated as the main results of this paper. System stability and performance analysis will be discussed in Section 4. Simulation results are proposed to demonstrate the merits of the proposed method in Section 5, and concluding remarks is placed in Section 6.

## 2. PROBLEM FORMULATION AND PRELIMINARIES

Multiple model ARDSC will be considered for the following semi-strict feedback nonlinear system.

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \phi_i^T(x_1, \cdots x_i, t)\,\theta_1 + d_1(x,t) \quad i = 1, \cdots, n-1 \\ \dot{x}_n &= u + \phi_n^T(x_1, \cdots, x_n, t)\,\theta_n + d_n(x,t) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^T$ is the state vector, $y \in R$ and $u \in R$ are the system output and input respectively, $\theta_i \in R^{p_i}, i = 1, \cdots, n$ represent the vectors of other unknown parameters, $p_i$ is the dimension of $\theta_i$, $d_i(x,t)$ is uncertain nonlinearity in the $i^{th}$ channel, and $\phi_i(x_1, \cdots, x_i, t) \in R^{p_i}$, $i = 1, \cdots, n$ are known smooth functions. Our purpose is to drive the output of system (1) to track the desired signal $x_d$ as closely as possible with reasonably good transient performance.



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To facilitate the control design, some necessary assumptions are given as follows.

*Assumption 1*
The extents of unknown parameter vectors are known, that is,

$$\theta_i \in \Omega_i = \{\theta_i \in R^{p_i} | \theta_{i\,\min} \leqslant \theta_i \leqslant \theta_{i\,\max}\}, \tag{2}$$

where $\theta_{i\,\min}, \theta_{i\,\max}, i = 1, \cdots, n$ are known constants.

*Assumption 2*
All the uncertain nonlinearities are bounded, that is, $|d_i(x,t)| \leqslant \delta_i, \quad i = 1, \cdots, n$, where $\delta_i$ are some positive constants.

*Assumption 3*
The states of system (1) are available, and the desired trajectory vectors are continuous and available, and $[x_d, \dot{x}_d, \ddot{x}_d]^T \in \Omega_d$ with a known compact set $\Omega_d = \left\{[x_d, \dot{x}_d, \ddot{x}_d]^T : x_d^2 + \dot{x}_d^2 + \ddot{x}_d^2 \leqslant B_0\right\} \subset R^3$, whose size $B_0$ is a known positive constant.

## 3. MULTIPLE MODEL ARDSC WITH PARAMETER RESETTING

### *3.1. Design of adaptive robust dynamic surface controllers*

A multiple-model ARDSC with estimator resetting is proposed in this note (Figure 1). The difference from the normal ARDSC is that $N$ fixed models and one identification model are ready as switching candidates in the parameter adaptation. Thus, faster parameter convergence and more accurate parameter estimates can be obtained, which will result in better transient performance of the tracking system.

From our previous work [6], the adaptive robust dynamic surface controller of system (1) can be given as follows.

$$u = \alpha_n = -\phi_n^T(x_1, \cdots, x_n)\,\hat{\theta}_n + \dot{x}_{nf} - k_{ns}z_n - z_{n-1} + \alpha_{ns2} \tag{3}$$

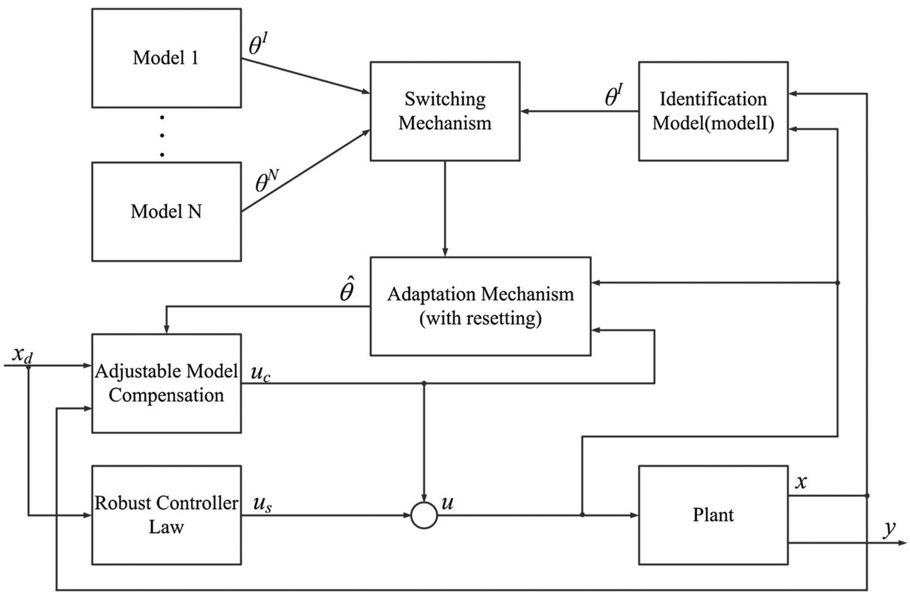


Figure 1. Diagram of control and parameter learning mechanism.

The control law $\alpha_n$ is recursively defined by

$$z_1 = x_1 - x_d, z_{i+1} = x_{i+1} - x_{(i+1)f}, \tag{4}$$

$$\begin{gathered} \dot{x}_{1f} = \dot{x}_d, \tau_{i+1}\dot{x}_{(i+1)f} + x_{(i+1)f} = \alpha_i - k_{(i+1)v}\tau_{i+1}\tanh\left(\frac{k_{(i+1)v}\tau_{i+1}y_{i+1}}{\varsigma_{i+1}}\right), \\ x_{(i+1)f}(0) = \alpha_i(0) \end{gathered} \tag{5}$$

$$\alpha_i = -\phi_i^T(x_1, \cdots, x_i)\hat{\theta}_i + \dot{x}_{if} - k_{is}z_i - z_{i-1} + \alpha_{is2}, \tag{6}$$

where $i = 1, \cdots, n-1$, $z_0 = 0$, $\hat{\theta}_i$ denotes the estimate of the unknown parameter by using switching and resetting update mechanism, $k_i$ is the feedback control gain to stabilize the nominal system, and $k_{v(i+1)}, \tau_{i+1}$ and $\varsigma_{i+1}$ are some positive design parameters. $\alpha_{is2}$ represents robust feedback to attenuate the effect of model uncertainties, which will be synthesized later.

The filter error is defined by

$$y_{i+1} = x_{(i+1)f} - \alpha_i. \tag{7}$$

Consequently, the error dynamics of the closed-loop system can be deduced as

$$\begin{cases} \dot{z}_i = \phi_i^T(\bar{x}_i, t)\tilde{\theta}_i + z_{i+1} + y_{i+1} + d_i + \alpha_{is2} - k_{is}z_i - z_{i-1} \\ \dot{z}_n = \phi_n^T(\bar{x}_n, t)\tilde{\theta}_n + d_n + \alpha_{ns2} - k_{ns}z_n - z_{n-1} \\ \dot{y}_{i+1} = -\frac{y_{i+1}}{\tau_{i+1}} - k_{(i+1)v}\tanh\left(\frac{k_{(i+1)v}y_{i+1}}{\zeta_{i+1}}\right) - \dot{\alpha}_i \\ i = 1, \cdots, n-1 \end{cases}. \tag{8}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ represents the estimated error of the unknown parameter $\theta_i$.

### 3.2. Determination of fixed models and identification model

From Assumption 1, we know every unknown parameter in $\theta_i$ has a fixed range, which is represented as $L_{\theta_{i,j}} = \theta_{i,j\max} - \theta_{i,j\min}$, $j = 1, 2, \cdots, p_i$. The simple way to determine the fixed models is to distribute the models evenly over the parameter space. The fixed width $L_{\theta_{i,j}}$ is split into $m_{\theta_{i,j}}$ equivalent parts. Then, we will have $m_{\theta_{i,j}} + 1$ fixed models for every parameter $\theta_{i,j}$. Thus, the number of the fixed models of the system is given by

$$N = \prod_{i=1}^{n}\prod_{j=1}^{p_i}\left(m_{\theta_{i,j}} + 1\right) \tag{9}$$

Theoretically, as long as the separation of each parameter is sufficiently small, a fast transient performance can be achieved because of the accurate parameter selection . However, the number of fixed models and computation load will increase exponentially along with the number of separation part. In this subsection, we try to construct an identification model as a special candidate fixed model to improve the transient performance with less number of fixed models. The following will demonstrate the construction detail of the identification model.

System (1) can be rewritten as the following compact form.

$$\dot{x} = f^T(x,t)\theta + g(x,u,t) + d(x,t) \tag{10}$$



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where

$$f^T(x,t) = \begin{bmatrix} \varphi_1^T & & & \\ & \varphi_2^T & & \\ & & \ddots & \\ & & & \varphi_n^T \end{bmatrix}, g(x,u,t) = \begin{bmatrix} x_2 \\ \vdots \\ x_n \\ u \end{bmatrix}$$

$$d(x,t) = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}, \theta = \left[\theta_1^T, \theta_2^T, \cdots, \theta_n^T\right]^T$$

Because $\dot{x}$ in (10) is unmeasured, we first introduce the following filters:

$$\dot{\omega}_0 = -k(t)\left(\omega_0 - x\right) + g(x,u,t), \tag{11}$$

$$\dot{\omega}^T = -k(t)\omega^T + f^T(x,t). \tag{12}$$

where $k(t) = k_0 + k_1(t),\ k_0 > 0, k_1(t) > 0$.

Defining

$$\zeta = x - \omega_0 - \omega^T\theta, \tag{13}$$

it follows from (10), (11), and (12), and $\zeta$ can be generated from

$$\dot{\zeta} = -k(t)\zeta + d(x,t). \tag{14}$$

Let

$$\begin{aligned} P &= \int_0^t \omega(t)\omega^T(t)dt, P(0) = P_0, \\ Q &= \int_0^t \omega(t)\left(x - \omega_0 - \zeta\right)dt, Q(0) = Q_0 \end{aligned}, \tag{15}$$

where $P_0, Q_0$ satisfy $P_0^{-1}Q_0 \in \Omega_\theta$.

*Definition 1 (*[18] *$\sigma$ degree of persistent excitation)*
When the determinant of $P$ is greater than $\sigma$, that is, $\det(P) > \sigma$, where $\sigma$ is some small positive constant to be designed, then we call $\sigma$ the degree of persistent excitation.

Remark: The degree $\sigma$ can be used to adjust the frequency of calculating the identification model. Small $\sigma$ can lead to a relatively frequent update of the identified model and yield a larger computation load. While large $\sigma$ will discourage the identification mode from following the real mode in time.

If the degree of persistent excitation condition is satisfied, the unknown parameter $\theta$ will be directly identified by

$$\theta = P^{-1}Q, \tag{16}$$

Noting Equation (14), the nonlinear uncertainty $d(x,t)$ is unknown. The signal $\zeta$ is generated by the following dynamic equation in implementation:

$$\dot{\zeta}' = -k(t)\zeta',\ \zeta'(0) = x(0) - \omega_0(0) - \omega^T(0)\theta \tag{17}$$



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Then the identification model is actually given as

$$\theta^I = P^{-1} Q' \tag{18}$$

where $Q' = \int_0^t \omega(t)\,(x - \omega_0 - \zeta')\,dt$, $Q'(0) = Q_0$.

Defining the parameter identification error $\tilde{\theta}^I = \theta - \theta^I$ and considering (14), (16), (17), and (18), we have

$$\tilde{\theta}^I = P^{-1} \int_0^t \omega(t) \tilde{\zeta}(t) dt, \tag{19}$$

where $\tilde{\zeta} = \zeta - \zeta'$ is the output of

$$\dot{\tilde{\zeta}} = -k(t)\tilde{\zeta} + d(x,t),\ \tilde{\zeta}(0) = 0. \tag{20}$$

*Remark 1*

The identification model can approximate the real model more accurately by increasing the gain $k(t)$. Moreover, if the nonlinear uncertainty $d_i = 0$, true parameter estimates can be calculated by (16). Thus, when the identification model is constructed successfully, even though the distribution for fixed models determination in the compact set $\Omega_\theta$, to which the plant parameter $\theta$ belongs, is relatively sparse, a good fixed model distribution can also be obtained.

### 3.3. Design of switching and parameter adaptation mechanisms

Define

$$\xi_j = x - \omega_0 - \omega^T \theta^j,\ j = 1, \cdots, N, N+1, N+2, \tag{21}$$

where $\theta^1, \cdots, \theta^N$ represent the parameter vectors of fixed models, $\theta^{N+1} = \theta^I$ is the parameter vector of identified model, and $\theta^{N+2}$ denotes the estimated vector in the adaptive model, that is, $\theta^{N+2} = \hat{\theta}$. Let $\{M_i\}_{i=1}^N$ denote the $N$ fixed models, $M_{N+1}$ denote the identification model, and $M_{N+2}$ denote the adaptive model. A rational performance criterion for switching is chosen as

$$J_j(t) = \alpha \xi_j^T(t) \xi_j(t) + \beta \int_0^t e^{-\lambda(t-\upsilon)} \xi_j^T(\upsilon) \xi_j(\upsilon) d\upsilon. \tag{22}$$

*Remark 2*

The index function defined in (22) can measure the approximation degree for the true model of the system. In general, the smallest value of $J_j$ represents that the model $j$ is closest to the real model.

***Switching mechanism**:* At first, preselect a positive number $T_{\min}$ and set $M_c = M_1$. The initial identification model is set as $\theta^I(0) = P_0^{-1} Q_0$. Then at each time $t > 0$, fixed models $\{M_i\}_{i=1}^N$, previous identification model $M_{N+1}\left(\theta^I\right)$, and adaptive model $M_{N+2}$ are the candidates to be switched. Set $M_{i*} = \underset{i=1,\cdots,N+2}{\arg\min} \{J_i(t)\}$. If in the ensuring $T_{\min}$ time interval, the $i^*$ does not change, then $M_c$ is switched to $M_{i*}$, and the parameter estimate vector is reset as $\hat{\theta}(t) = \theta^{i^*}$.

If the PE condition is satisfied at time $t_i$ where $i = 1, 2, \cdots$, let a persistent variable $a$ be updated by $a = P^{-1} Q'$, and $P(t_i)$, $Q'(t_i)$ are reset as $P(t_i) = 0$, $Q'(t_i) = 0$. When the latest switching is finished, that is, time interval $T_{\min}$ is over, the parameter vector of identification model is updated by $\theta^I = a$. The process is then repeated. The switching mechanism in flowchart is shown in Figure 2.

 
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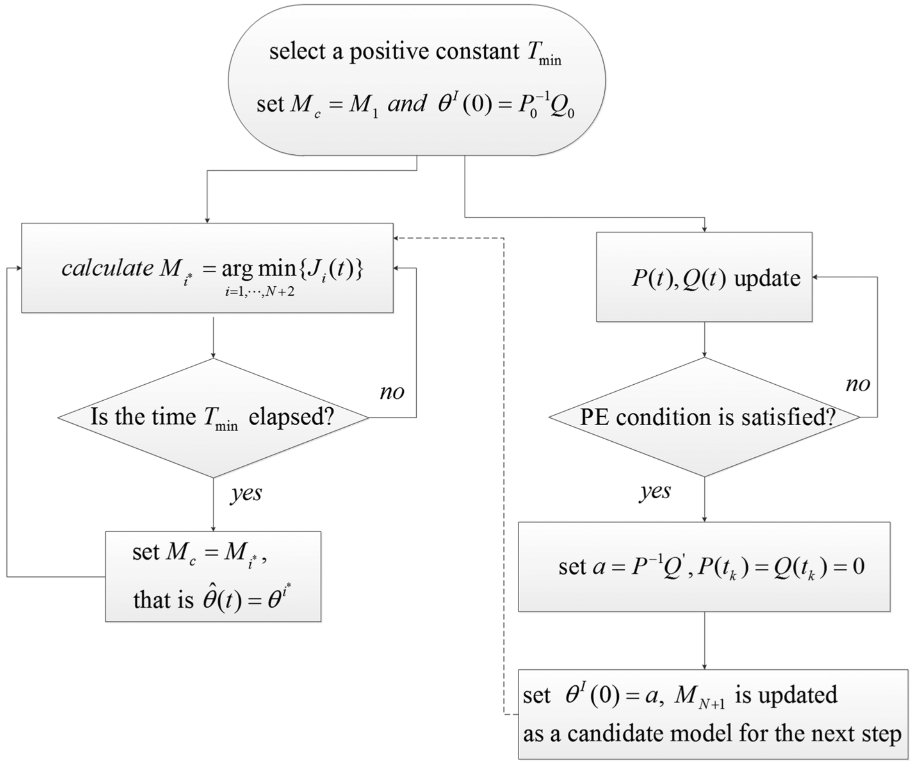


Figure 2. Switching mechanism in flowchart.

*Remark 3*
In the switching procedure, we know that the identification model (a special fixed model) can be updated according to the PE condition to catch the actual model of the system. Even though possible parameter jumps, the identification will still catch the real model as soon as possible. By using the resetting method, most of history information can be cut off. Thus, more robust identification model can be obtained, and the large value of partial elements in the persistent matrices can be avoided.

***Parameter update mechanism**:* Suppose $t_k, k \in \{1, 2, \cdots\}$ is the $k^{th}$ switching time instance when the parameter estimation is reset. The parameter update mechanism is given as follows.

$$\begin{cases} \hat{\theta}(t) = \theta^{i^*}(t), \dot{\hat{\theta}}_a(t) = 0 \quad t = t_k, k = 1, 2, \cdots \\ \hat{\theta}(t) = \hat{\theta}_a(t), \dot{\hat{\theta}}_a(t) = \text{ Proj } (\Gamma\pi), \; t \in (t_k, t_{k+1}) \end{cases}. \tag{23}$$

where $\Gamma = diag\,(\Gamma_1, \cdots, \Gamma_n)$, $\pi = \left[z_1\phi_1^T\,(\bar{x}_1)\,, \cdots, z_n\phi_n^T\,(\bar{x}_n)\right]^T$,
$\text{Proj}(\Gamma\pi) = \left[\text{Proj}_{\hat{\theta}_1}(\bullet), \cdots, \text{Proj}_{\hat{\theta}_M}(\bullet)\right]^T$, $M = \sum_{i=1}^{n} p_i$, $\bullet$ represents the $j^{th}$ element of vector $\Gamma\pi$. The projection map has the following expression and properties.

$$\text{Proj}_{\hat{\theta}_j}(\bullet) = \begin{cases} 0 \;\text{ if }\; \theta_j = \theta_{j_{\max}} \;\text{ and }\; \bullet > 0 \\ 0 \;\text{ if }\; \theta_j = \theta_{j_{\min}} \;\text{ and }\; \bullet < 0 \\ \bullet \qquad\qquad \text{otherwise} \end{cases} \tag{24}$$

The footnote $j \in \{1, 2, \cdots, M\}$ denotes the $j^{th}$ element. $j_{\min}, j_{\max}$ denote the minimum and maximum values of the $j^{th}$ element of $\theta$, respectively. It can be shown that for any adaptation function $\pi$, the projection mapping guarantees



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$$\text{Property1}: \quad \hat{\theta} \in \Omega_\theta = \left\{ \hat{\theta} : \theta_{\min} \leqslant \hat{\theta} \leqslant \theta_{\max} \right\}; \tag{25}$$

$$\text{Property2}: \quad \tilde{\theta}^T \left( \pi - \Gamma^{-1} \operatorname{Proj}_{\hat{\theta}}(\Gamma \pi) \right) \leqslant 0, \quad \forall \pi. \tag{26}$$

where $\theta_{\min} = \left[ \theta_{1_{\min}}^T, \cdots, \theta_{n_{\min}}^T \right]^T$, $\theta_{\max} = \left[ \theta_{1_{\max}}^T, \cdots, \theta_{n_{\max}}^T \right]^T$.

## 4. PERFORMANCE ANALYSIS OF THE CLOSED-LOOP SYSTEM

From the construction of fixed models and identification model described in the aforementioned subsection, we explicitly have

$$\tilde{\theta}(t) \in \Omega_\theta, t \geqslant 0. \tag{27}$$

where the parameter estimation error is bounded by $\left\| \tilde{\theta} \right\| \leqslant \| \theta_{\max} - \theta_{\min} \|$. As a result, the robust term $\alpha_{is2}$ in the designed controller (6) can be selected as

$$\alpha_{is2} = h_i \tanh\left( \frac{h_i z_i}{\varepsilon_i} \right), \tag{28}$$

where $h_i$ satisfies

$$h_i \geqslant \sup_{\theta_i \in \Omega_{\theta_i}} \left| \phi_i^T (\bar{x}_i, t) \tilde{\theta}_i + d_i(x, t) \right|. \tag{29}$$

Then, we have

$$z_i \left( \phi_i^T (\bar{x}_i, t) \tilde{\theta}_i + d_i(x, t) + \alpha_{is2} \right) \leqslant 0.2785 \varepsilon_i. \tag{30}$$

*Theorem 1*
Consider the nonlinear system given in (1) with parametric and nonlinear uncertainties. Given any positive constant $p$, for all the initial condition of the closed-loop system satisfying $\sum_{i=1}^{n} z_i^2(0) + \sum_{i=1}^{n-1} y_{i+1}^2(0) \leqslant p$, the initial estimates of the unknown parameters being selected such that $\hat{\theta}_i(0) \in \Omega_i$ and the controller being given by (3), there exist control parameters $k_i, \tau_{i+1}, k_{(i+1)v}$ such that the closed-loop system (8) is semi-globally uniformly ultimately bounded stable under any parameter update law, which can make the parameter estimates stay in the compact set $\Omega_\theta$.

*Proof*
Define a positive definite function $V_s$ as

$$V_s = \frac{1}{2} \sum_{i=1}^{n} z_i^2 + \frac{1}{2} \sum_{i=1}^{n-1} y_{i+1}^2. \tag{31}$$

Differentiating function $V_s$ with respect to time $t$, one has

$$\begin{aligned} \dot{V}_s = & \sum_{i=1}^{n-1} \left\{ z_i \left( \varphi_i^T (\bar{x}_i, t) \tilde{\theta}_i + z_{i+1} + y_{i+1} + d_i + \alpha_{is2} - k_{is} z_i - z_{i-1} \right) \right\} \\ & + z_n \left( \varphi_n^T (\bar{x}_n, t) \tilde{\theta}_n + d_n + \alpha_{ns2} - k_{ns} z_n - z_{n-1} \right) \\ & + \sum_{i=1}^{n-1} \left\{ -\frac{y_{i+1}^2}{\tau_{i+1}} - k_{(i+1)v} y_{i+1} \tanh\left( \frac{k_{(i+1)v} y_{i+1}}{\zeta_{i+1}} \right) - y_{i+1} \dot{\alpha}_i \right\} \end{aligned} \tag{32}$$



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The right hand of Equation (32) can be simplified as

$$\begin{aligned}\dot{V}_s = &\sum_{i=1}^{n}\left\{z_i\left(\varphi_i^T\left(\bar{x}_i,t\right)\tilde{\theta}_i + d_i + \alpha_{is2}\right\} - \sum_{i=1}^{n}k_{is}z_i^2 + \sum_{i=1}^{n-1}z_i y_{i+1} \\ &+ \sum_{i=1}^{n-1}\left\{-\frac{y_{i+1}^2}{\tau_{i+1}} - k_{(i+1)v}y_{i+1}\tanh\left(\frac{k_{(i+1)v}y_{i+1}}{\zeta_{i+1}}\right) - y_{i+1}\dot{\alpha}_i\right\}\end{aligned} \tag{33}$$

No matter what the parameter update law is adopted, if the parameter estimate $\hat{\theta}(t)$ does not escape from the range of $\Omega_\theta$, the inequality (30) can hold. Using the facts

$$z_i y_{i+1} \leqslant z_i^2 + \frac{1}{4}y_{i+1}^2$$

$$-y_{i+1}\dot{\alpha}_i \leqslant |\dot{\alpha}_i|\,|y_{i+1}|,$$

one has

$$\begin{aligned}\dot{V}_s \leqslant\ &0.2785\sum_{i=1}^{n}\varepsilon_i - \sum_{i=1}^{n-1}\left\{(k_{is}-1)z_i^2 + \left(\frac{1}{\tau_{i+1}} - \frac{1}{4}\right)y_{i+1}^2\right\} - k_{ns}z_n^2 \\ &+ \sum_{i=1}^{n-1}\left\{|\dot{\alpha}_i|\,|y_{i+1}| - k_{(i+1)v}y_{i+1}\tanh\left(\frac{k_{(i+1)v}y_{i+1}}{\zeta_{i+1}}\right)\right\}\end{aligned} \tag{34}$$

If the control parameters $k_i, \tau_{i+1}, k_{(i+1)v}$ satisfy

$$\begin{cases} k_{is} \geqslant \kappa + 1, \quad k_{ns} \geqslant \kappa \\ \frac{1}{\tau_{i+1}} \geqslant \kappa + \frac{1}{4}, \quad k_{(i+1)v} \geqslant M_i \\ i = 1, 2, \cdots, n-1 \end{cases}, \tag{35}$$

where $M_i$ is the bound of $|\dot{\alpha}|$ and noting the following established inequality

$$|\dot{\alpha}_i|\,|y_{i+1}| - k_{(i+1)v}y_{i+1}sign\left(y_{i+1}\right) \leqslant 0, \tag{36}$$

$$\left|k_{(i+1)v}y_{i+1}\right| - k_{(i+1)v}y_{i+1}\tanh\left(\frac{k_{(i+1)v}y_{i+1}}{\zeta_{i+1}}\right) \leqslant 0.2785\zeta_{i+1}, \tag{37}$$

then, we have

$$\dot{V}_s \leqslant -2\kappa V_s + \mu. \tag{38}$$

which leads to the following inequality

$$0 \leqslant V_s(t) \leqslant \exp(-2\kappa t)V_s(0) + \frac{\mu}{2\kappa}[1 - \exp(-2\kappa t)], \tag{39}$$

where

$$\mu = 0.2785\left(\sum_{i=1}^{n}\varepsilon_i + \sum_{i=1}^{n-1}\zeta_{i+1}\right). \tag{40}$$

Thus, we conclude that the close-loop system (8) is uniformly ultimately bounded stable. Because the bound of $|\dot{\alpha}|$ is required during the proof procedure, only semi-globally stability is achieved. □



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*Remark 4*
Theorem 1 indicates that as long as the parameter update law can make the parameter estimates stay in the compact set $\Omega_\theta$, the tracking system is stable, which means the design of the parameter adaptation mechanism is separated from the controller design. Thus, the gradient update law derived from Lyapunov stability analysis in the normal ARC is not the only choice. Better parameter convergence will be available through using some advanced learning mechanisms (e.g., least square method, multiple identifiers, and estimator resetting methods) as long as the bound of parameter error $\tilde{\theta}$ can be guaranteed.

*Proposition 1*
If the parameter update law is chosen as (23), better transient performance can be obtained, and more accurate tracking of the system can be obtained when the PE condition is satisfied comparing with that using the normal adaptive robust dynamic surface control.

*Proof*
The following inequality can be deduced by invoking the inequality (30):

$$\begin{aligned}\dot{V}_s &\leqslant Z^T\tilde{\theta}^T f(x,t) + Z^T D_1 + Z^T u_{s2} - \kappa\|Z\|^2 - \kappa\|Y\|^2 + \mu_1 \\ &\leqslant -(\kappa-1)\|Z\|^2 - \kappa\|Y\|^2 - \sum_{i=1}^{n} h_i|z_i| + \left\|\tilde{\theta}^T\right\|\left\|Z^T f(x,t)\right\| + \frac{1}{4}\|D_1\|^2 + \mu\end{aligned} \tag{41}$$

where $Z = [z_1, z_2, \cdots, z_n]^T \subset R^n$, $Y = [y_2, \cdots, y_n]^T \subset R^{n-1}$, $D_1 = [d_1, \cdots, d_n]^T$, $u_{s2} = [u_{1s2}, \cdots, u_{ns2}]^T$, $\mu_1 = 0.2785\sum_{i=1}^{n-1}\zeta_{i+1}$.

From (41), we know that in order to obtain better tracking performance, the positive term $\frac{1}{4}\|D_1\|^2 + \left\|\tilde{\theta}^T\right\|\left\|Z^T f(x,t)\right\| + \mu$ should decreased quickly. In the proposed method, the parameter update law is chosen as (23). The switching criterion defined in (22) is dependent on which model is closer to the real model. Thus, when the switching occurs, $\tilde{\theta}$ obviously approaches zero faster than $\tilde{\theta}_a$ in the normal ARC. Subsequently, the derivative of $V_s$ will have more negative value that produces faster tracking convergence. Moreover, if the identification model is obtained, more accurate parameter estimates will be obtained, and this results in a decrease in the tracking error. □

## 5. EXAMPLES

Consider the following nonlinear system in semi-strict feedback form:

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta_1^T\phi_1(x_1) + d_1(x,t) \\ \dot{x}_2 &= u + \theta_2^T\phi_2(x_1,x_2) + d_2(x,t) \\ y &= x_1\end{aligned} \tag{42}$$

where $\theta_1 = [0.5, 2]^T$, $\theta_2 = [1.2, 1]^T$, $\phi_1(x_1) = \left[x_1^2, \sin(x_1)\right]^T$, $\phi_2(x_1,x_2) = [x_1\sin(x_2), x_2]^T$, $d_1(x,t) = 0.1\sin(10\pi t)$, $d_2(x,t) = 0.2f(x_1)$ and where $f(x_1) = \begin{cases}0, & x_1 < 0, \\ x_1, & 0 \leqslant x_1 \leqslant 1, \\ 1, & x_1 > 1.\end{cases}$ .The extents of the unknown parameters are supposed as $\theta_{11} \in (0,2)$, $\theta_{12} \in (0,5)$, $\theta_{21} \in (0,4)$, $\theta_{22} \in (0,3)$. Every parameter extent is divided into four equal parts, and we take the cut-point as a fixed model. For example, divide the parameter $\theta_{11}$ into four equal parts with the cut-points 1,0.5,1.5. Thus, for every parameter, the fixed models are determined as $\theta_{11} = \{0.5, \ 1, \ 1.5\}$, $\theta_{12} = \{1.25, \ 2.5, \ 3.75\}$, $\theta_{21} = \{1, \ 2, \ 3\}$, $\theta_{22} = \{0.75, \ 1.5, \ 2.25\}$. So the total number of fixed models is $3 \times 3 \times 3 \times 3 = 81$. The initial values of the parameter and state estimates are set as zero. The initial states are supposed as $x_1(0) = 0$, $x_2(0) = 0$. The control parameters are chosen as $k_1 = 2, k_2 = 5, \tau_2 = 0.2, k_{2v} = 10, \varsigma_2 = 0.001$,



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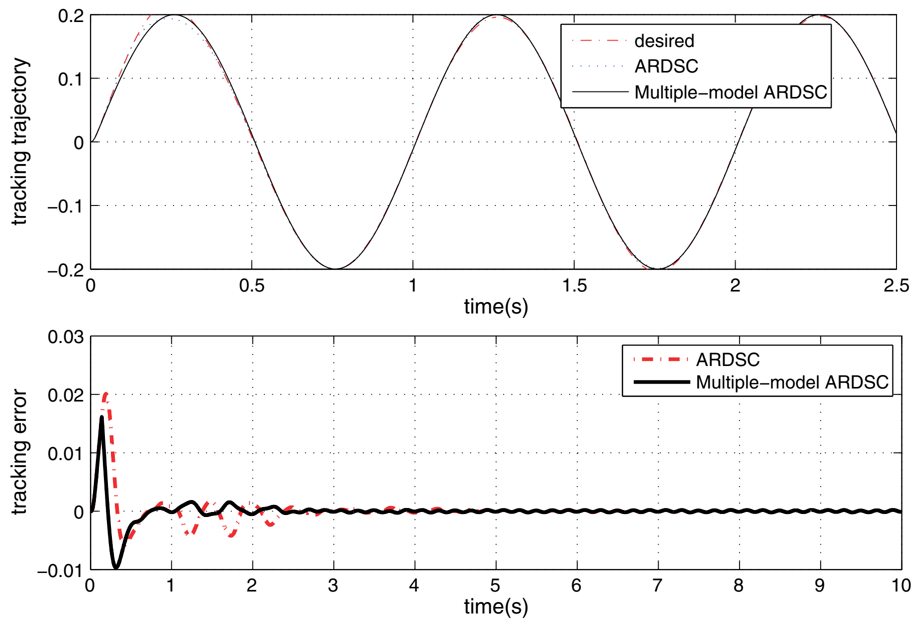


Figure 3. Tracking trajectories and errors of adaptive robust dynamic surface control (ARDSC) and multiple-model ARDSC.

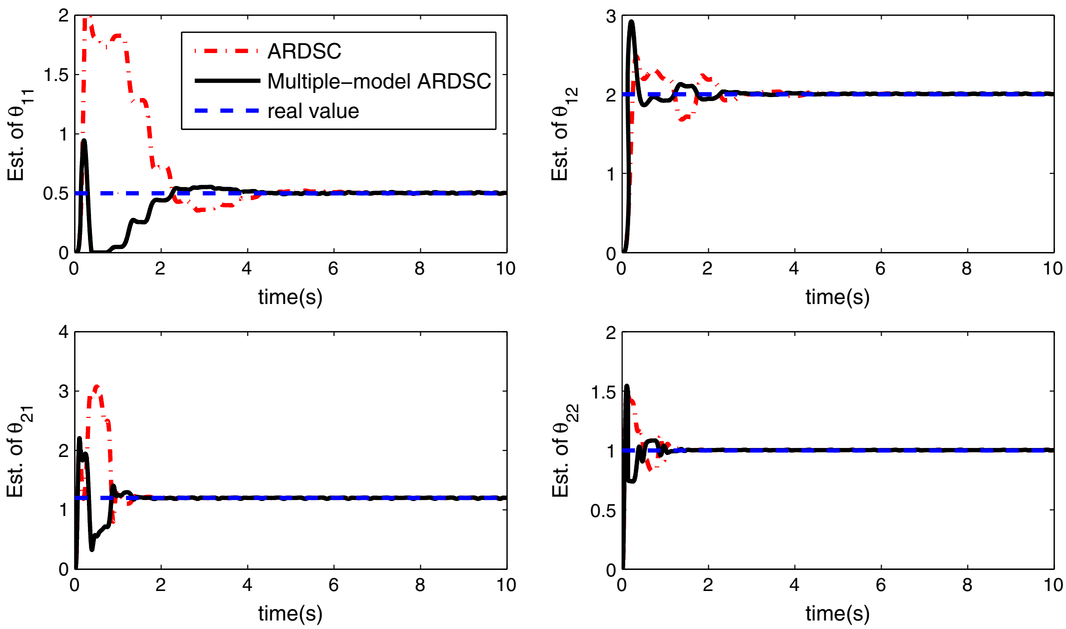


Figure 4. Parameter estimates.

$h_1 = 20, h_2 = 40$, $\varepsilon_1 = \varepsilon_2 = 10$. Adaptation gain matrix is given by $\Gamma = 80 \times diag(1500, 100, 100, 7)$. The the degree $\sigma = 10^{-8}$ and $\gamma = 2$, $\kappa = 0.5$, $T_{min} = 50ms$. The desired trajectory is chosen as $x_d(t) = 0.2sin(2\pi t)$. We consider the following cases to demonstrate the effectiveness of multiple-model ARDSC.

**Case 1: ARDSC and multiple-model ARDSC without the identifier model**

In Figure 3, the steady tracking errors are both well, while the transient performance of multiple-model ARDSC is much better. Figure 4 shows that using the multiple-model approach brings better parameter estimation performance (transient performance and final estimation accuracy). In Figure 5, it definitely shows that at the initial system operation, the switching action is effective, and there is no switching action after about 1 s. Thus, after about 1 s, only the adaptive model is effective.

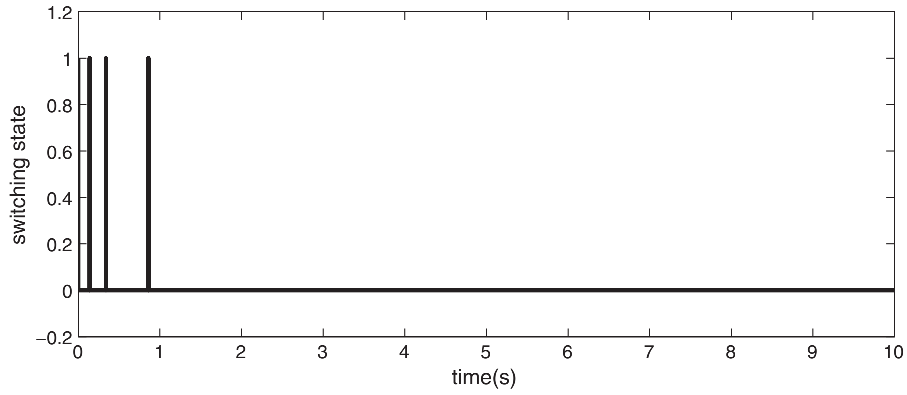


Figure 5. Switching state in Case 1.

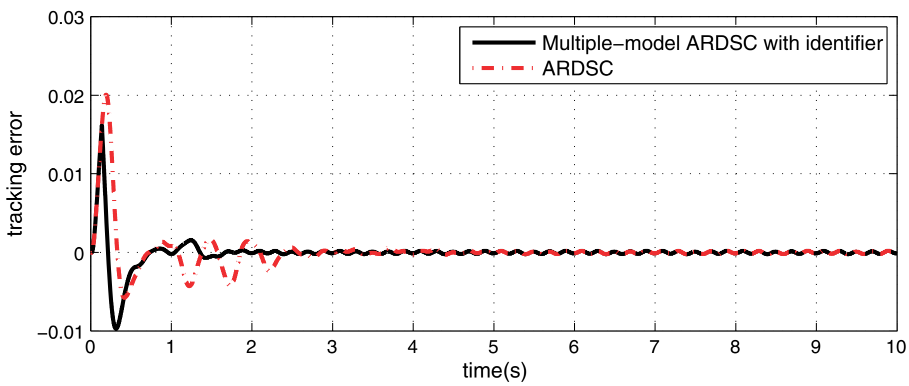


Figure 6. Output tracking errors in Case 2.

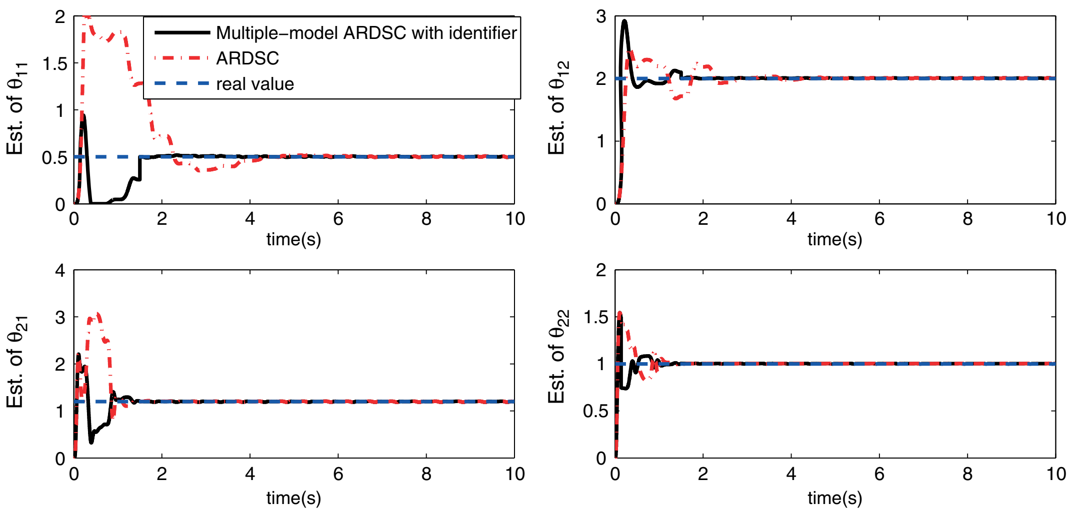


Figure 7. Parameter estimates in Case 2.



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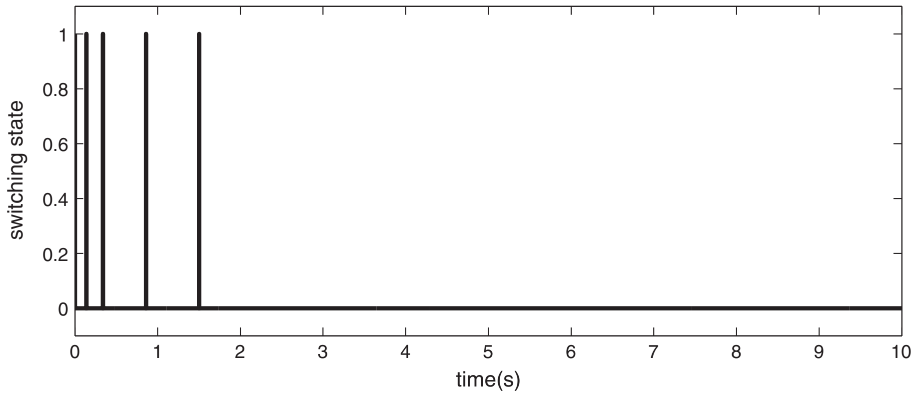


Figure 8. Switching state in Case 2.

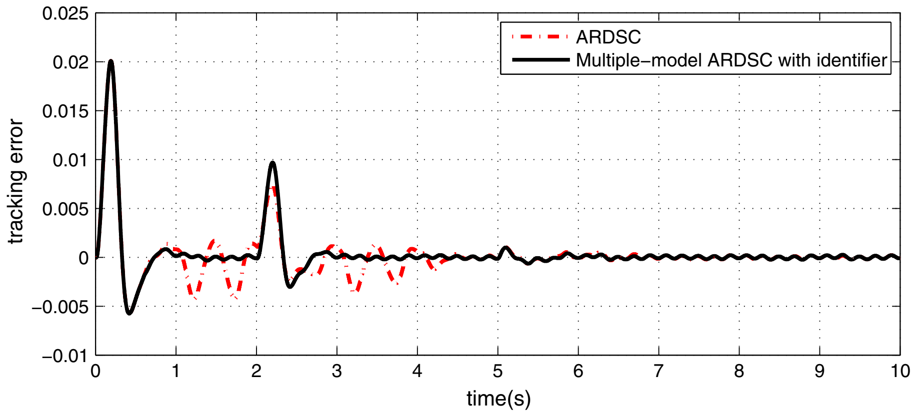


Figure 9. Output tracking errors in Case 3.

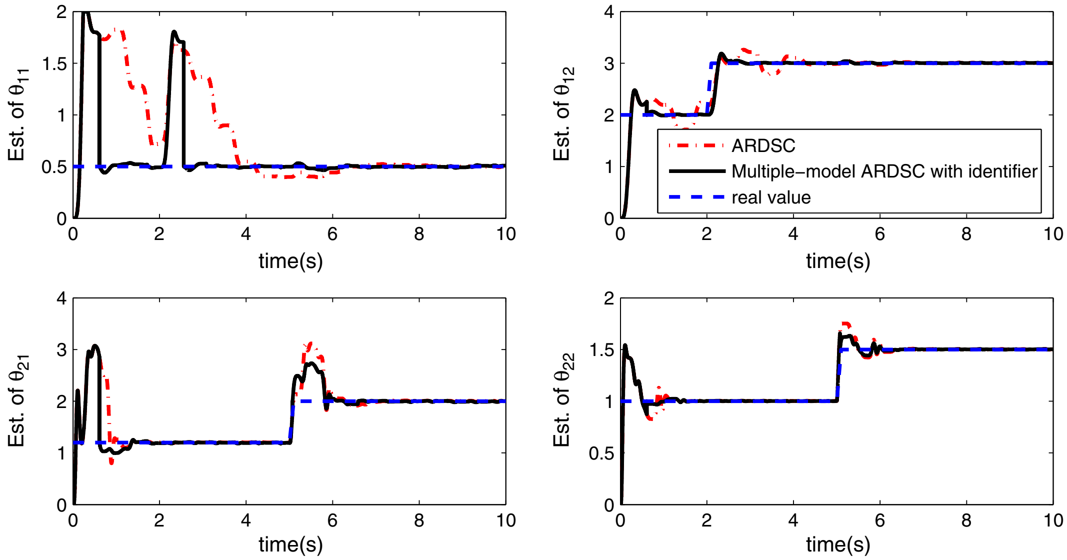


Figure 10. Parameter estimates in Case 3.



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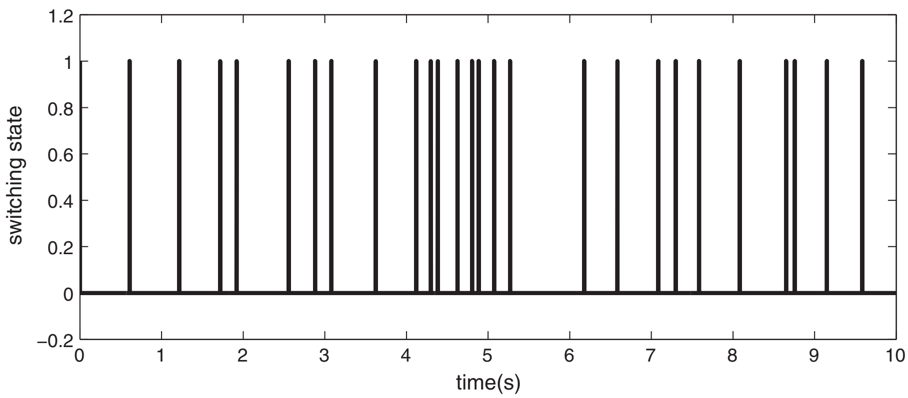


Figure 11. Switching state in Case 3.

**Case 2: ARDSC and multiple-model ARDSC with the identifier model and the fixed model being halved**

Figure 6 shows that even the number of fixed models is halved, the tracking error of multiple-model ARDSC converges to a small value with better transient performance. The parameter estimates jump to the actual values when the identifier model is obtained. As shown in Figure 7, the estimate is switched to the identifier model at about 2 s. Comparing with Figures 8 and 6, the switching is more frequent in Case 2.

**Case 3: ARDSC and multiple-model ARDSC considering jump parameters**

Suppose that the unknown parameters in system (42) jumps as follows:

$$\Theta(t) = \begin{cases} [0.5, 2, 1.2, 1]^T, & t \leqslant 2, \\ [0.5, 3, 1.2, 1]^T, & 2 < t \leqslant 5, \\ [0.5, 3, 2, 1.5]^T, & t > 5 \end{cases}$$

In Figure 9, multiple-model ARDSC shows better transient performance and steady state accuracy than those of ARDSC. Figure 10 implies that owing to employing the identifier model, the parameter estimates can catch the actual values even there exists a parameter jumping in the system operation. Figure 11 shows the switching profile.

## 6. CONCLUSIONS

A multiple-model ARC has been investigated in this paper. It is different from the traditional multiple-model adaptive controller because an identification model was employed as a special fixed model. Thus, when the PE condition is satisfied, the accurate model can be obtained, and better transient performance of the parameter estimates would be obtained with relatively few number of fixed models. The advancement of the proposed method is demonstrated by the simulation results.

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